Assignment Feedback in School Choice Mechanisms*

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Abstract

This paper experimentally investigates the provision of real-time feedback about school assignments during the preference reporting period in three widely employed mechanisms: deferred acceptance, top trading cycles, and the Boston mechanism. Adaptive models predict that greater sensitivity to tentative assignments during the reporting period will produce more equilibrium assignments in all three mechanisms. Consistent with adaptive predictions, real-time assignment feedback consistently increased equilibrium assignments but did not increase truthful reporting. These findings suggest that providing feedback about assignments during the preference reporting period could help student assignment mechanisms more reliably achieve policy goals.

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1 Introduction

Children in the United States are traditionally assigned to public schools based on where they live, but a growing number of school districts now allow parents to indicate their preferences over schools. In these districts, policy makers employ mechanisms that assign students to schools based on reported preferences and legally determined priorities. Participants can often submit and adjust their preference reports over a period of several days. School assignments are typically revealed after the end of the reporting period when all preference reports are finalized.

This paper experimentally investigates whether real-time feedback about tentative assignments produces more frequent equilibrium outcomes than providing information only at the end of the reporting period in three student assignment mechanisms: deferred acceptance, top trading cycles, and the Boston mechanism. This hypothesis is motivated by adaptive models that describe dynamic behavioral adjustment according to simple rules. Adaptive models predict that all three mechanisms will achieve equilibrium assignments more frequently when participants are sensitive to tentative assignments during the preference reporting period.

Consistent with adaptive predictions, real-time assignment feedback produced more equilibrium assignments in all three mechanisms. Consistent with equilibrium predictions, real-time assignment feedback increased efficiency in top trading cycles and it eliminated more justified envy in the deferred acceptance mechanism. No mechanism can guarantee both Pareto efficiency and the elimination of justified envy in equilibrium, so different mechanisms are designed to achieve different policy goals. The dominant-strategy equilibrium of the top trading cycles mechanism achieves Pareto efficiency, while the dominant-strategy equilibrium of the deferred acceptance mechanism eliminates justified envy. All Nash equilibria of the Boston mechanism eliminate justified envy.

While truth-telling is a major objective in the design of school choice mechanisms, real-time feedback did not increase the frequency of truthful reporting. Truthful reporting is a weakly dominant strategy in both top trading cycles and deferred acceptance. In contrast, the Boston mechanism has no dominant strategy. In all three mechanisms, inaccurate preference reports often produce

the same assignments as truthful preference reports. Consequently, all three mechanisms have Nash equilibria involving inaccurate preference reports and adaptive models predict that sensitivity to tentative assignments is insufficient to reliably produce truthful preference reports.

Historically, calculating mechanism assignments was time consuming and computationally expensive, so assignment feedback was rarely provided during the reporting period. Technological advances have significantly reduced these obstacles. Many school districts already utilize online interfaces for on-demand preference reporting. The Wake County public school system provided real-time feedback about the first choices of other participants in a Boston mechanism (Dur et al., 2018). Inner Mongolia provided feedback in a dynamic queuing mechanism where participants apply to one school at a time (Gong and Liang, 2020). The results of this paper suggest that providing feedback about tentative assignments during the reporting period could help school choice mechanisms more reliably achieve policy goals.

The remainder of this paper is organized as follows. Section 2 discusses the related literature. Section 3 presents the theory. Section 4 describes the experimental design. Section 5 covers the hypotheses. Section 6 presents the results and section 7 concludes.

2 Related Literature

This paper experimentally investigates real-time assignment feedback in student assignment mechanisms where participants report preference rankings over schools. In contrast, several previous studies (Chen and Sönmez, 2006; Pais and Pintér, 2008; Calsamiglia et al., 2010; Klijn et al., 2013; Featherstone and Niederle, 2016; Chen et al., 2018; Ding and Schotter, 2019; Chen and Kesten, 2019) experimentally investigate student assignment mechanisms that only provide feedback at the end of the reporting period. Table 1 highlights key differences between previous experimental studies and the present paper.

Klijn et al. (2019) and Bó and Hakimov (2020) experimentally investigate iterative versions of the deferred acceptance mechanism where participants apply to one school at a time. Instead of reporting a ranking over schools, partici-

	Real-Time Feedback	Real-Time Assignment Feedback	Participants Report Rankings
Chen and Sönmez (2006)	✗	✗	✓
Pais and Pintér (2008)	✗	✗	✓
Calsamiglia et al. (2010)	✗	✗	✓
Klijn et al. (2013)	✗	✗	✓
Featherstone and Niederle (2016)	✗	✗	✓
Chen et al. (2018)	✗	✗	✓
Guillen and Hakimov (2018)	✗	✗	✓
Ding and Schotter (2019)	✗	✗	✓
Chen and Kesten (2019)	✗	✗	✓
Dur et al. (2018)	✓	✗	✓
Gong and Liang (2020)	✓	✓	✗
Klijn et al. (2019)	✓	✓	✗
Bó and Hakimov (2020)	✓	✓	✗
This Paper	✓	✓	✓

Table 1: School Choice Matching Experiments

pants in these mechanisms applied to one school at a time during each step of an iterative process that mirrors the algorithm performed by the conventional deferred acceptance mechanism. At the end of each step, participants received feedback about whether their application was accepted or rejected. Rejected applicants could apply elsewhere during the next step. In contrast, the present paper considers mechanisms where participants report rankings over schools rather than applying to only one school at a time.

Gong and Liang (2020) investigate a dynamic queuing mechanism under which participants select one school at a time and are free to adjust their selection until the reporting period ends. During the reporting period, participants received feedback about the quotas for each school and the standardized test scores of applicants to each school. In this setting, all schools have identical preferences which rank students according to standardized test scores. At the end of the reporting period, each school accepts applicants in rank order up to its quota and rejects the remaining applicants. Rationalizable strategy profiles are shown to produce stable outcomes in this mechanism, but it is not strategy proof as it has no dominant strategy. In contrast, the present paper considers strategy-proof mechanisms where participants report rankings over

schools rather than selecting only one school at a time.

Dur et al. (2018) investigate a variant of the Boston mechanism employed by the Wake County Public School System in North Carolina. Participants in this mechanism could log in to a website and adjust their preference report at any time during a two-week period. Each time a participant visited the website, they received feedback about how many others ranked each school as their top choice. In contrast, the present paper investigates feedback about tentative assignments rather than feedback about top choices.

Guillen and Hakimov (2018) experimentally investigate the provision of top-down advice in the top trading cycles mechanism. They find that subjects in the top trading cycles mechanism truthfully reported their top choice more often when they received top-down advice describing the strategy-proofness of the mechanism. The present paper finds that subjects in the top trading cycles mechanism truthfully reported their top choice more often when they received assignment feedback during the preference reporting period. However, it also finds that subjects did not truthfully report their full preference ranking more often when they received assignment feedback.

The hypotheses tested by this paper are motivated by adaptive models. Oprea et al. (2011) experimentally investigate adaptive models in Hawk-Dove games. Cason et al. (2013) test adaptive predictions in rock-paper-scissors. Stephenson (2019) investigate adaptive models in attacker-defender games. Stephenson and Brown (2020) investigate adaptive models in all-pay auctions. Schneider and Stephenson (2021) test adaptive models in markets with asymmetric information. In contrast, the present paper tests adaptive models in student assignment mechanisms.

The present paper investigates three widely employed mechanisms: the deferred acceptance mechanism, the top trading cycles mechanism, and the Boston mechanism. Gale and Shapley (1962) describe the deferred acceptance mechanism. Shapley and Scarf (1974) describe the top trading cycles mechanism. Ergin and Sönmez (2006) characterize Nash equilibria of the Boston mechanism. Abdulkadiroglu and Sönmez (2003) discuss the fundamental trade-off between Pareto efficiency and the elimination of justified envy in school choice settings.

3 Theory

Student assignment mechanisms face a fundamental trade-off between Pareto efficiency and the elimination of justified envy. Section 3.1 illustrates this trade-off and describes the market structure used in the experiment. Section 3.2 describes the mechanisms. Section 3.3 considers Nash equilibria. Section 3.4 discusses adaptive models.

3.1 Market Structure

Consider a setting[1] where each of three schools can accept up to m students and each student can be assigned to only one school. There are three types of students and m students of each type for a total of $n = 3m$ students. Each student has strict preferences over schools and each school has a strict priority ranking over students.

Student Type	1	2	3
	b	a	a
Preference	a	b	b
	c	c	c

School	a	b	c
	1	2	2
Priority	3	1	1
	2	3	3

School a always gives higher priority to type 1 students than type 3 students and always gives higher priority to type 3 students than type 2 students. Priority rankings between students of same type are determined by lottery. This market structure will be used as a running example throughout the paper.

A student x is said to have justified envy towards another student y if x prefers the school assigned to y and x also has a higher priority than y at this school. If no student has justified envy under a particular assignment we say that this assignment eliminates justified envy. In this market structure, the only assignment that eliminates justified envy is given by

$$\mu = \begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$$

[1] A similar setting with only one student of each type is considered by Roth (1982) and Abdulkadiroglu and Sönmez (2003).

Here all type 1 students are assigned to school a, all type 2 students are assigned to school b, and all type 3 students are assigned to school c. Yet μ is Pareto dominated by the assignment

$$\lambda = \begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix}$$

Here all type 1 students are assigned to school b, all type 2 students are assigned to school a, and all type 3 students are assigned to school c. The assignment λ Pareto dominates the assignment μ because type 1 students prefer b over a and type 2 student prefer a over b.

The assignment λ is Pareto optimal but it fails to fully eliminate justified envy because it gives type 3 students justified envy towards type 2 students. Type 3 students would prefer school a over school c, and school a ranks type 3 students higher than type 2 students. The assignment μ uniquely eliminates justified envy but is Pareto dominated by λ, so no Pareto optimal assignment can eliminate justified envy in this market structure.

3.2 Student Assignment Mechanisms

Student assignment mechanisms assign students to schools based on reported student preferences over schools and student priorities at each school. This paper considers three widely employed mechanisms: the Boston mechanism, the top trading cycles mechanism, and the deferred acceptance mechanism. Top trading cycles and deferred acceptance are both strategy proof. The dominant-strategy Nash equilibrium of the top-trading cycles mechanism always achieves Pareto efficiency. The dominant-strategy Nash equilibrium of the deferred acceptance mechanism always eliminates justified envy. The Boston mechanism is manipulable, but its Nash equilibria always eliminate justified envy. The following paragraphs describe the algorithm performed by each mechanism.

3.2.1 The Boston Mechanism

Under the Boston mechanism, each student initially applies to her top choice of schools according to her reported preferences. Each school accepts applicants

in priority order until it runs out of seats. The remaining students apply to their second choice of schools according to their reported preferences. Again, each school accepts students in priority order until it runs out of seats. This process repeats until every student is assigned to a school. If students report truthfully, the Boston mechanism will select a Pareto optimal assignment. However, students can often benefit by misreporting their preferences. Ergin and Sönmez (2006) show that the set of Nash equilibrium assignments for the Boston Mechanism coincide exactly with the set of assignments that eliminate justified envy under the true preferences.

3.2.2 The Deferred Acceptance Mechanism

Under the student optimal deferred acceptance mechanism, each student initially applies to her top choice of schools according to her reported preferences. Each school tentatively accepts applicants in priority order until it runs out of seats. The remaining applications are rejected. Students whose applications were rejected then apply to their next highest choice of schools. Each school then considers its new applicants alongside those it has already tentatively accepted. It tentatively accepts its top priority students among this group until it run out of seats and rejects the remaining students. This process repeats until every student is assigned to a school.

3.2.3 The Top Trading Cycles Mechanism

The top trading cycles mechanism constructs a directed graph based the priorities and reported preferences. Each school points to it's highest priority student and each student points to her most preferred school according to her reported preferences. Since there are a finite number of schools and students, the directed graph will have at least one cycle. Students who are part of a cycle are assigned to the school they point at. Each of the remaining students point to their most preferred school according to their reported preferences among those schools that still have open seats. Each school points to their highest priority student among those students that remain unassigned. Students who are part of a cycle are assigned to the school they point to. This process repeats until every student is assigned to a school.

3.3 Nash Equilibria

Consider the market structure described in section 3.1 with $n = 24$ students. The truthful report for type 1 students is (b, a, c) while the truthful report for other students is (a, b, c). If everyone reports truthfully, deferred acceptance will select the assignment μ and top trading cycles will select the assignment λ. In strategy proof mechanisms, truthful preference reports are often *weakly* dominant but not *strictly* dominant. Inaccurate preference reports frequently yield the same assignments as truthful preference reports. In the deferred acceptance mechanism, if type 1 and type 2 students report truthfully then type 3 students will be assigned to school c regardless of their preference report. In the top trading cycles mechanism, if type 2 and type 3 students report truthfully then type 1 students will be assigned to school b as long as they report it as their top choice.

In strategy proof mechanisms, truthful reports always form a Nash equilibrium, but other Nash equilibria often involve inaccurate reports. For example, the report profile under which type 3 students send the inaccurate report (b, a, c) and other students report truthfully is a Nash equilibrium in both deferred acceptance and top trading cycles. Like the truthful report profile, it produces the assignment μ under deferred acceptance and the assignment λ under top trading cycles.

Deferred acceptance and top trading cycles also have Nash equilibria that produce different assignments than the truthful report profile. For example, the report profile under which type 3 students send the inaccurate report (b, c, a) while other students report truthfully is a Nash equilibrium that produces the assignment λ under both mechanisms. The report profile under which type 1 students send the inaccurate report (a, b, c) and type 2 students send the inaccurate report (b, a, c) while type 3 students report truthfully is a Nash equilibrium that produces the assignment μ under both mechanisms.

Table 2 summarizes the aforementioned Nash equilibria. All report profiles listed in this figure are Nash equilibria of both deferred acceptance and top trading cycles. The first row describes the truthful report profile. The second row describes an inaccurate report profile that leaves the assignments unchanged from the truthful report profile. The third row describes an inac-

Nash Equilibrium Preference Reports			Assignment	
			Deferred	Top Trading
Type 1	Type 2	Type 3	Acceptance	Cycles
(b, a, c)	(a, b, c)	(a, b, c)	μ	λ
(b, a, c)	(a, b, c)	(b, a, c)	μ	λ
(b, a, c)	(a, b, c)	(b, c, a)	λ	λ
(a, b, c)	(b, a, c)	(a, b, c)	μ	μ

Table 2: Multiple Nash Equilibria in Strategy Proof Mechanisms

curate report profile that produces the assignment λ under both mechanisms. The fourth row describes an inaccurate report profile that produces the assignment μ under both mechanisms.

A Nash equilibrium said to be neutrally stable[2] if a unilateral deviation by one agent never creates an incentive for another agent to deviate. More precisely, a report profile r is neutrally stable if for all participants $i, j \in N$ and all reports $q_i, q_j \in R$ we have $\pi_i(r_i, q_j, r_{-i,j}) \geq \pi_i(q_i, q_j, r_{-i,j})$. All neutrally stable equilibria of the deferred acceptance mechanism yield the dominant-strategy equilibrium assignment μ. All neutrally stable equilibria of the top trading cycles mechanism yield the dominant-strategy equilibrium assignment λ. The Boston mechanism has no neutrally stable Nash equilibria. A complete list of symmetric Nash equilibria and their stability properties is provided in appendix A.

3.4 Adaptive Models

Adaptive models describe agents who adjust their behavior over time according to simple rules (Fudenberg et al., 1998; Hofbauer and Sigmund, 2003; Sandholm, 2010). Different adaptive models have different informational requirements. The replicator dynamic (Taylor and Jonker, 1978) and the Brown-Von Neumann-Nash (BNN) dynamic (Brown and Von Neumann, 1950) require agents to utilize information about the actions and payoffs of others. In the present setting, agents can only observe their own actions and payoffs, so this

[2]This definition of neutral stability is closely related to that of Smith (1982). The present definition applies to finite populations while that of Smith (1982) applies to infinite populations.

section focuses on adaptive models under which agents only need information about their own actions and payoffs. The best response dynamic (Cournot, 1838; Gilboa and Matsui, 1991) describes agents who select actions to myopically maximize their payoffs. The logit dynamic (Fudenberg et al., 1998) describes agents who are more likely to select actions with higher payoffs.

In student assignment mechanisms, a population of n agents adjust their preference reports over a finite period spanning the time interval $[0, T] \subseteq \mathbb{R}_+$. Let $N = \{1, 2, \ldots, n\}$ denote the set of agents. Let X denote the set of feasible preference reports. Let $r_i^t \in X$ denote the report selected by agent i at time $t \in [0, T]$. Assignments are determined by the finalized reports $r_i^T \in X$ selected at the end of the reporting period. Let $\pi_i(x)$ denote agent i's payoff from her assignment under the report profile $x = (x_1, x_2, \ldots, x_n) \in X^n$.

$$\pi_i(x) = \begin{cases} 3 & \text{if agent } i \text{ is assigned to her favorite school} \\ 2 & \text{if agent } i \text{ is assigned to her second favorite school} \\ 1 & \text{if agent } i \text{ is assigned to her least favorite school} \end{cases}$$

A report profile $x \in X^n$ is said to be a Nash equilibrium if each agent simultaneously best responds to the reports selected by others such that

$$x_i \in \operatorname*{argmax}_{y_i \in X} \pi_i(y_i, x_{-i}) \quad \text{for all } i \in N \tag{1}$$

A report $x_i \in X$ is said to be weakly dominant for agent i if $\pi_i(x_i, y_{-i}) \geq \pi_i(y_i, y_{-i})$ for all $y \in X^n$. A report profile $x \in X^n$ is said to be a dominant-strategy equilibrium if $\pi_i(x_i, y_{-i}) \geq \pi_i(y_i, y_{-i})$ for all $i \in N$ and all $y \in X^n$. Let $\pi_i^t(x_i)$ denote agent i's payoff from her assignment under the report profile $\left(x_i, r_{-i}^t\right) \in X^n$.

$$\pi_i^t(x_i) = \pi_i\left(x_i, r_{-i}^t\right) \tag{2}$$

Let r_i^* denote agent i's truthful report. Let δ denote the Kronecker delta.

$$\delta(x, y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{if } x \neq y \end{cases} \tag{3}$$

Let $a_i^t(x_i)$ denote agent i's attraction to report $x_i \in X$ at time t.

$$a_i^t(x_i) = \alpha\pi_i^t(x_i) + \beta\delta(x_i, r_i^*) + \gamma\delta\left(x_i, r_i^t\right) \tag{4}$$

The parameter α denotes sensitivity to tentative assignments during the reporting period. The parameter β denotes the tendency for agents to select truthful reports more often than inaccurate reports. The parameter γ denotes the strength of behavioral inertia. Behavioral inertia is the tendency for agents to continue doing what they did in the past (Norman, 2009; Liu and Riyanto, 2017). Each agent revises her report at points in time generated by an independent Poisson process. Let $P_i^t(x_i)$ denote the probability that an agent i who revises her report at time t selects the new report x_i.

$$P_i^t(x_i) = \frac{\exp\left(a_i^t(x_i)\right)}{\sum_{y_i \in X} \exp\left(a_i^t(y_i)\right)} \tag{5}$$

Taking the limit as $\alpha \to \infty$ obtains a model under which agents always select payoff maximizing reports. Taking the limit as $\beta \to \infty$ obtains a model under which agents always select truthful reports. Taking the limit as $\gamma \to \infty$ obtains a model under which agents never change their reports. If $\gamma = \beta = 0$ as $\alpha \to \infty$ then agents select a maximizer at random, so agents can always deviate from non-strict Nash equilibria. Larger values of γ make agents less likely to switch between maximizers. Neutrally stable Nash equilibria are less susceptible to such deviations because unilateral deviations by one agent never create an incentive for other agents to deviate. If $\gamma = \sqrt{\alpha}$ as $\alpha \to \infty$ then agents never switch between maximizers and every Nash equilibrium is a fixed point.

Figures 1, 2, and 3 illustrate the mean path of adaptive models in top trading cycles, deferred acceptance, and the Boston mechanism respectively. Horizontal axes indicate time within the reporting period. For top trading cycles and deferred acceptance, the vertical axes in the first two rows indicate the percentage of dominant-strategy equilibrium assignments. For the Boston mechanism, the vertical axes in the first two rows indicate the percentage of Nash equilibrium assignments. The vertical axes in the last two rows indicate the percentage of truthful preference reports.

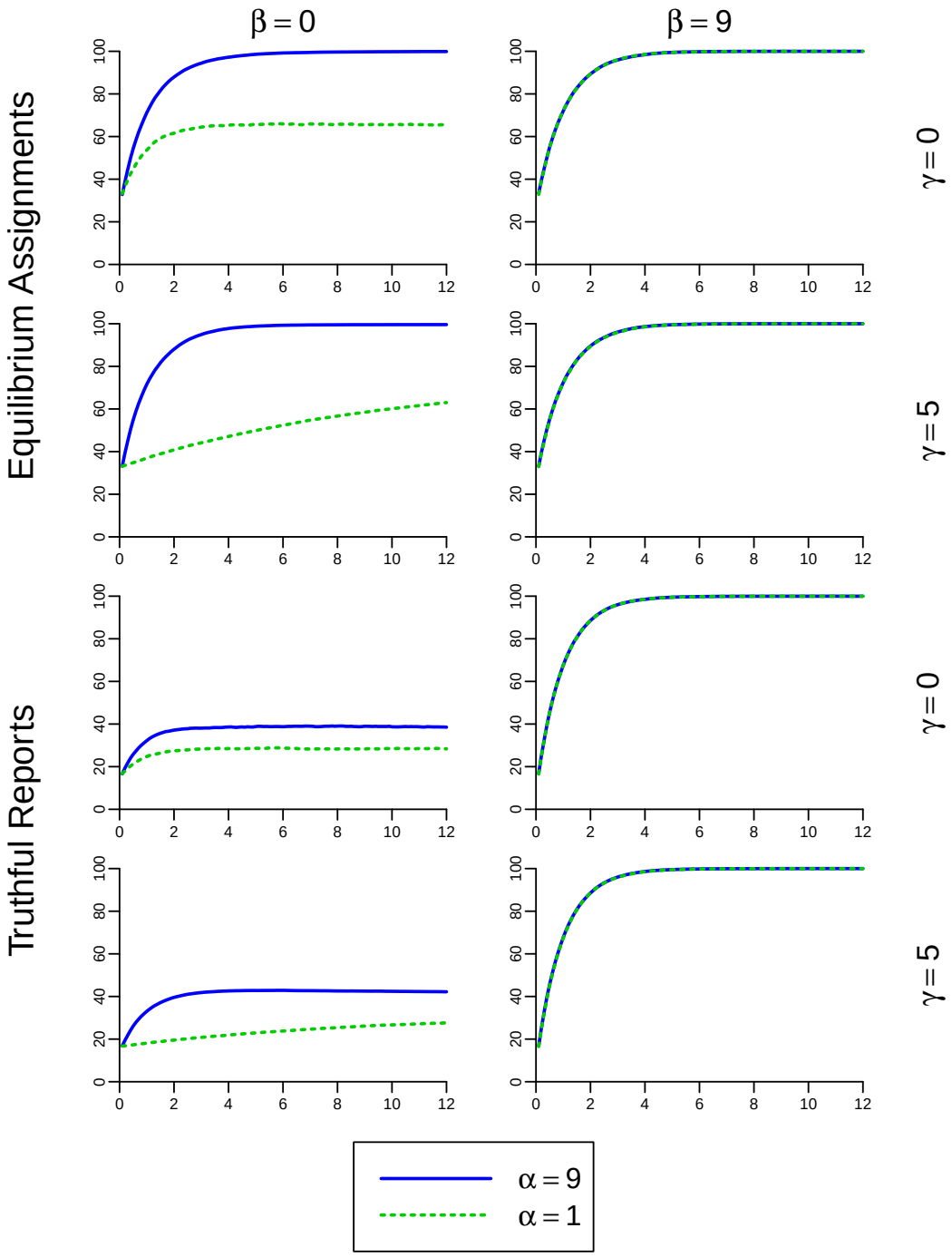

Figure 1: Adaptive mean path predictions in the top trading cycles mechanism. The horizontal axis indicates time within the reporting period. The vertical axis in the first two rows indicates the percentage of dominant-strategy equilibrium assignments. The vertical axis in the last two rows indicates the percentage of truthful preference reports.

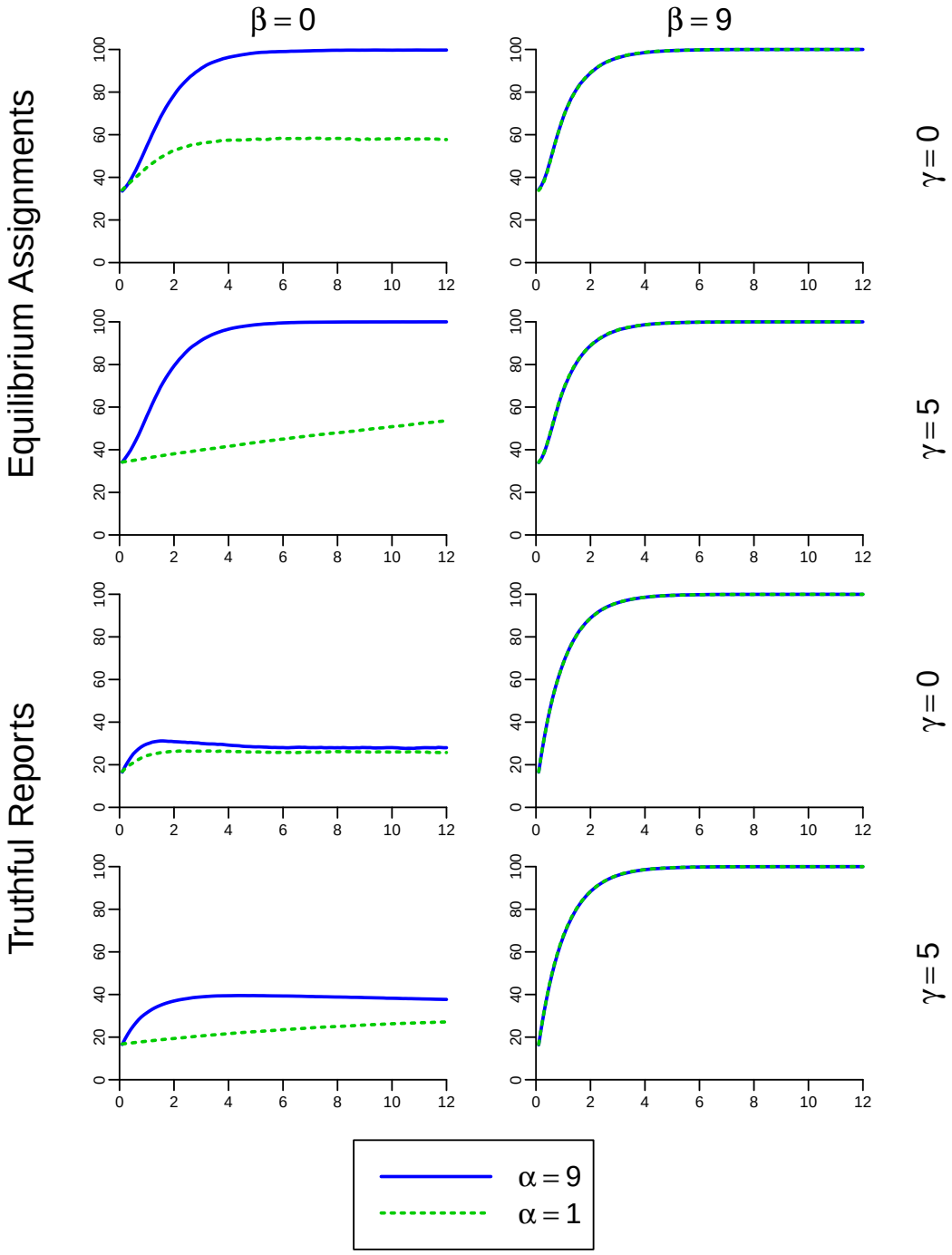

Figure 2: Adaptive mean path predictions in the deferred acceptance mechanism. The horizontal axis indicates time within the reporting period. The vertical axis in the first two rows indicates the percentage of dominant-strategy equilibrium assignments. The vertical axis in the last two rows indicates the percentage of truthful preference reports.

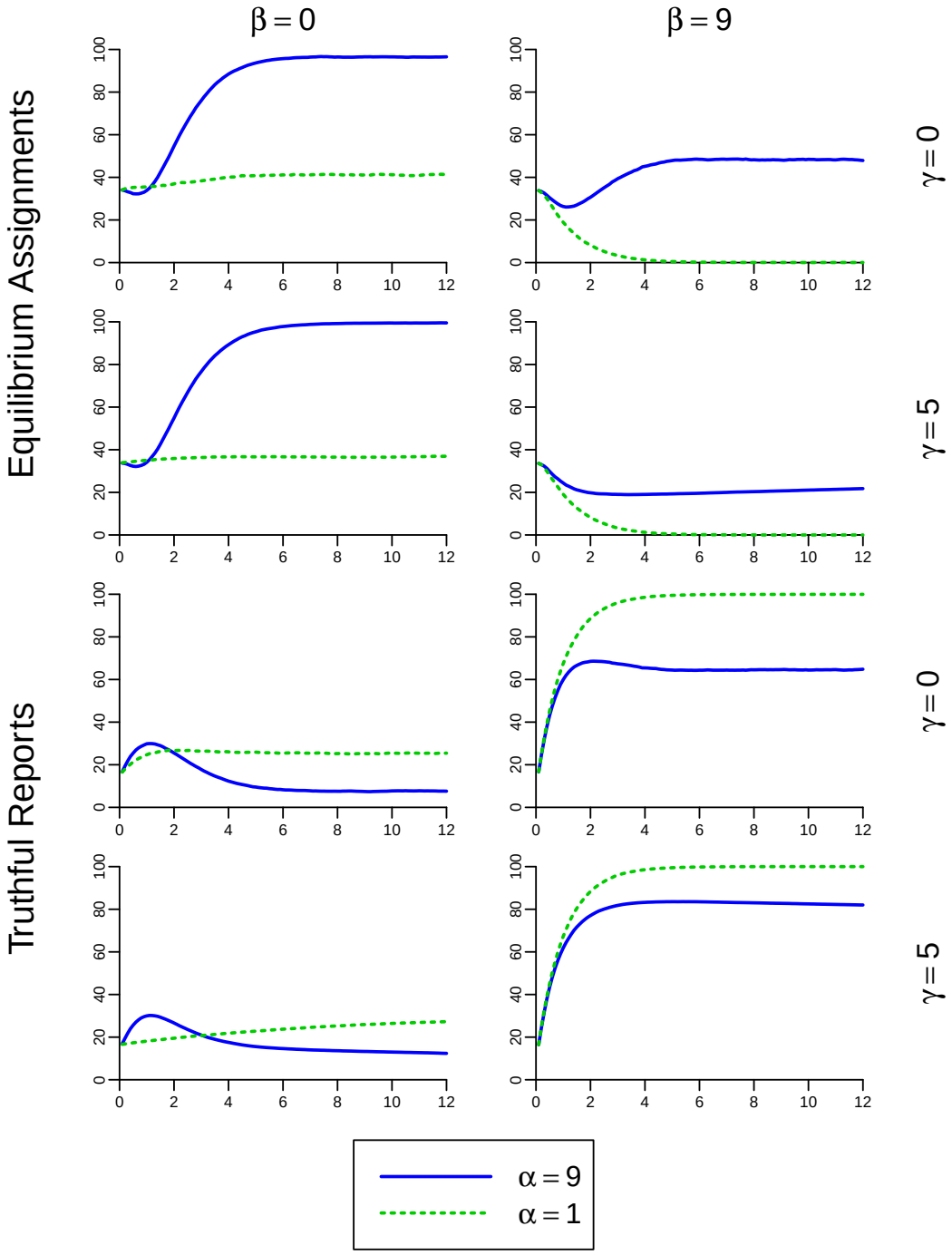

Figure 3: Adaptive mean path predictions in the Boston mechanism. The horizontal axis indicates time within the reporting period. The vertical axis in the first two rows indicates the percentage of Nash equilibrium assignments. The vertical axis in the last two rows indicates the percentage of truthful preference reports.

Truthful reporting is a dominant strategy in top trading cycles and deferred acceptance. As discussed in section 3.3, all neutrally stable Nash equilibria of these mechanisms produce dominant-strategy equilibrium assignments. Accordingly, higher values of α produce more dominant-strategy equilibrium assignments in both strategy-proof mechanisms. Greater sensitivity to tentative assignments during the reporting period produces more Nash equilibrium assignments in the Boston mechanism. All three mechanisms exhibit an initial period of convergence followed by relative stability. Initial convergence occurs as agents switch to preference reports that produce more desirable school assignments. Once the report profile approximates a Nash equilibrium, agents have less incentive to adjust their reports.

In top trading cycles and deferred acceptance, truth telling is weakly dominant but not strictly dominant. Inaccurate reports often produce the same assignments as truthful reports. In the deferred acceptance mechanism, if type 1 and type 2 students report truthfully then type 3 students are assigned to school c regardless of their preference report. In the top trading cycles mechanism, if type 2 students and type 3 students report truthfully then type 1 students are assigned to school b as long as they list it as their first choice. Since agents can best respond without reporting truthfully, sensitivity to tentative assignments is insufficient to reliably induce truthful preference reports. If truthful reporting was strictly dominant, then sensitivity to tentative assignments would reliably induce truthful preference reports.

Both top trading cycles and deferred acceptance achieve dominant-strategy equilibrium assignments more frequently when agents report truthfully. In the Boston mechanism, Nash equilibria generally involve inaccurate preference reports. Consequently, the Boston mechanism achieves fewer equilibrium assignments when agents exhibit a stronger tendency to select truthful reports and it produces fewer truthful reports when agents exhibit greater sensitivity to tentative assignments. As discussed in section 3.3, the Boston mechanism has no neutrally stable Nash equilibria. Consequently, in the absence of behavioral inertia, sensitivity to tentative assignments is insufficient for complete convergence to Nash equilibrium assignments in the Boston mechanism.

Adaptive models predict that increasing sensitivity to tentative assignments brings assignments more in line with equilibrium predictions. No mecha-

nism can guarantee both Pareto efficiency and the elimination of justified envy, so different mechanisms are designed to achieve different goals. The dominant-strategy equilibrium of top trading cycles achieves Pareto efficiency, so adaptive models predict that the top trading cycles will achieve greater efficiency when participants are more sensitive to tentative assignments. The dominant-strategy equilibrium of deferred acceptance mechanism eliminates justified envy, so adaptive models predict that deferred acceptance will eliminate more justified envy when participants are more sensitive to tentative assignments.

4 Experimental Design and Procedures

This study implements a 2×3 experimental design with six treatment conditions illustrated by table 3. Each experimental session implemented one of the six treatment conditions. A total of 18 experimental sessions were conducted, three for each of the six treatment conditions. All sessions were conducted at the Texas A&M University Economic Research Laboratory. Participants were recruited from the subject-pool of undergraduate students at Texas A&M University. Each session had 24 subjects for a total of 432 experimental subjects. Each experimental session consisted of 12 reporting periods and each reporting period lasted for exactly one minute. At the beginning of each session, subjects participated in one practice period that did not effect their earnings.

Each reporting period implemented a single school choice mechanism in which all 24 subjects participated. The market structure is described by section 3.1. Student assignment mechanisms face a fundamental trade-off between Pareto efficiency and the elimination of justified envy. No mechanism can guarantee both Pareto efficiency and the elimination of justified envy. The market structure described in section 3.1 is ideal for investigating this trade-off because assignments that eliminate justified envy are Pareto dominated and Pareto efficient assignments fail to eliminate justified envy.

At the beginning of each reporting period, subjects were randomly assigned one of the three student types described in section 3.1. Each type was assigned to 8 participants. Types were randomly reassigned at the beginning of each

	Top Trading Cycles	Deferred Acceptance	Boston
Discrete Feedback	3 Sessions	3 Sessions	3 Sessions
Real-Time Feedback	3 Sessions	3 Sessions	3 Sessions

Table 3: 2×3 Experimental Design with 18 Sessions

reporting period. Subjects were informed about the earnings they would receive from each of three possible options: a, b, or c. This information remained visible to subjects throughout the reporting period. To avoid introducing a psychological labeling or ordering bias, the labeling for each option and the order in which the options were presented was randomly reassigned at the beginning of each period.

Subjects could freely adjust their preference reports during the preference reporting period in every experimental treatment. In the discrete feedback treatment, subjects were only informed about their assignment at the end of each period, after all preference reports had been finalized. In the real-time feedback treatment, subjects also received real-time feedback about their tentative assignments during the preference reporting period. At the end of each session, subjects received their average earnings over all twelve periods plus a five dollar participation bonus. Average earnings were \$24.78 per subject.

Figure 4 depicts the experimental interface. The first column depicts the interface during the reporting period. The second column depicts the interface at the end of the reporting period. The first row depicts the interface in the discrete feedback treatment. The second row depicts the interface under the real-time feedback treatment. Subjects in the discrete feedback treatment could only observe their assignment at the end of the reporting period. Subjects in the real-time feedback treatment could also observe their tentative assignments throughout the reporting period.

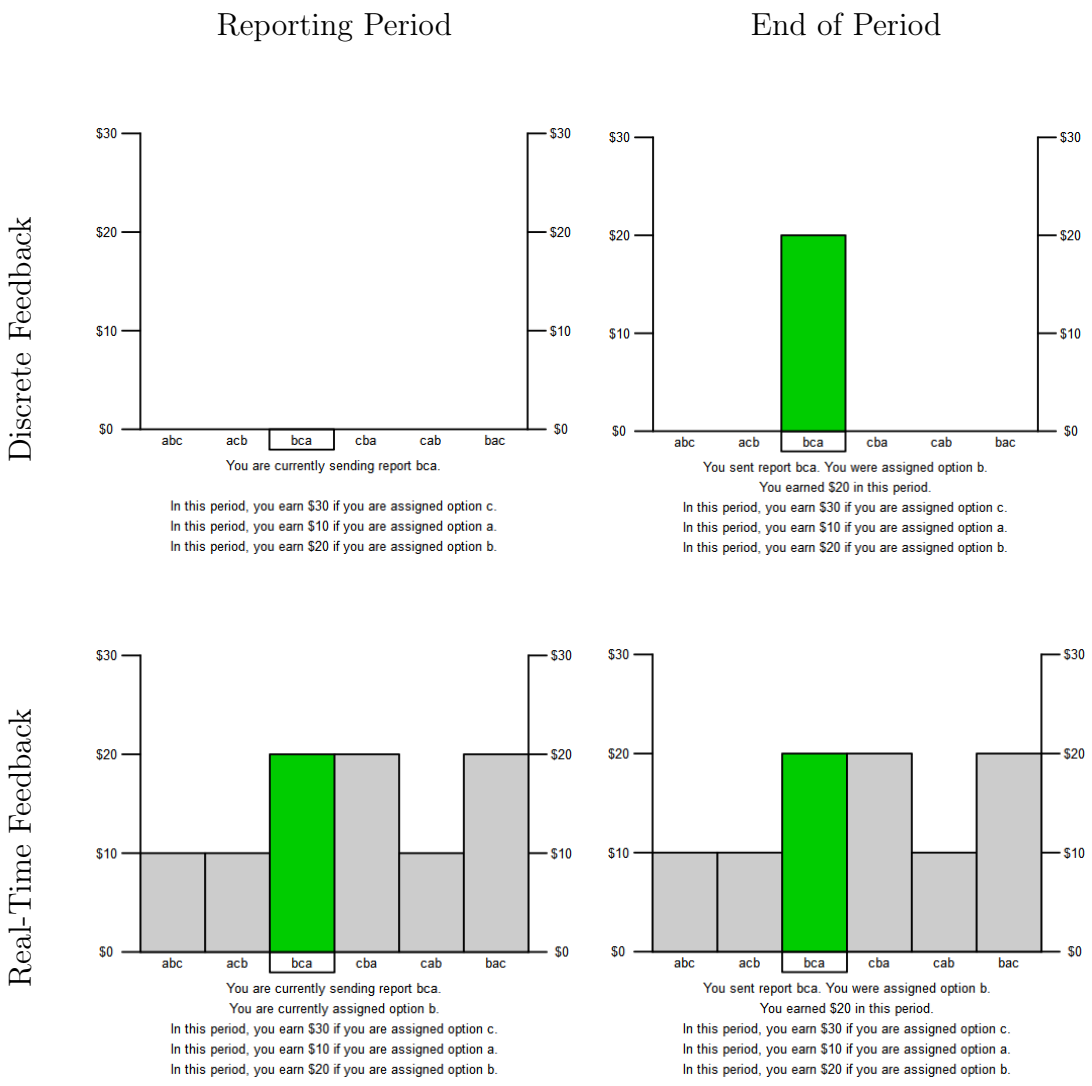

Figure 4: Screenshots of the Experimental Interface

5 Hypotheses

Adaptive models describe the dynamic behavior of agents who adjust their behavior over time according to simple rules. They predict that increased sensitivity to tentative assignments during the reporting period will produce more equilibrium assignments in all three mechanisms. Top trading cycles and deferred acceptance are strategy-proof. In these mechanisms, greater sensitivity to tentative assignments is predicted to produce more dominant-strategy equilibrium assignments. The Boston mechanism is manipulable and it has no dominant strategy. Greater sensitivity to tentative assignments is predicted to produce more Nash equilibrium assignments in the Boston mechanism.

Hypothesis 1. *All three mechanisms will achieve more equilibrium assignments under real-time feedback than discrete feedback.*

The dominant-strategy equilibrium of the deferred acceptance mechanism fully eliminates justified envy. All Nash equilibria of the Boston mechanism fully eliminate justified envy. In contrast, the dominant strategy Nash equilibrium of the top trading cycles mechanism does not fully eliminate justified envy.

Hypothesis 2. *The deferred acceptance and Boston mechanisms will eliminate more justified envy under real-time feedback than discrete feedback.*

The dominant-strategy Nash equilibrium of the top trading cycles mechanism assigns type 1 and type 2 students to their most preferred school. School a is most preferred by type 2 and type 3 students, but it only has room for one third of the student population. Consequently, it is impossible to assign more than two thirds of the student population to their most preferred school. The dominant-strategy Nash equilibrium of the top trading cycles mechanism achieves this upper bound. In contrast, neither the Nash equilibria of the Boston mechanism nor the dominant-strategy equilibrium of the deferred acceptance mechanism assign any students to their most preferred school in this market structure.

Hypothesis 3. *The top trading cycles mechanism will assign more students to their most preferred school under real-time feedback than discrete feedback.*

In strategy-proof mechanisms, truthful reporting is often weakly dominant but not strictly dominant. Inaccurate preference reports can yield the same assignment as truthful preference reports. Consequently, the experimental market structure produces multiple Nash equilibria in all three mechanisms. Since participants can learn to optimize their reports without reporting truthfully, adaptive models predict that sensitivity to assignments is insufficient to reliably produce truthful preference reports.

6 Results

Figures 5 and 6 illustrate average behavior by treatment. The horizontal axis in figure 5 indicates seconds within the reporting period. The horizontal axis in figure 6 indicates reporting periods within the session. In both figures, the first row depicts the top trading cycles mechanism, the second row depicts the deferred acceptance mechanism, and the third row depicts the Boston mechanism. For top trading cycles and deferred acceptance, the left column illustrates the percentage of dominant-strategy equilibrium assignments. The Boston mechanism has no dominant strategy. For the Boston mechanism, the left column illustrates the percentage of Nash equilibrium assignments. The right column illustrates the percentage of truthful preference reports.

Tables 4-7 provide treatment level averages and rank-sum tests. Each hypothesis test compares the 9 discrete feedback sessions with the 9 real-time feedback sessions for total of 18 observations stratified by mechanism accordingly to the rank-sum test of Zhao (2006) and Van Elteren (1960). Table 4 provides the percentage of subjects who selected a best response. A rank-sum test finds these values to be significantly different across treatments at the 1% level.

Table 5 provides the percentage of Nash equilibrium assignments in the Boston mechanism and the percentage of dominant-strategy equilibrium assignments in the other two mechanisms. In top trading cycles and deferred acceptance, adaptive models predict that greater sensitivity to tentative assignments will produce more dominant-strategy equilibrium assignments. Consistent with adaptive model predictions, real-time assignment feedback produced more dominant-strategy equilibrium assignments in these mechanisms. The Boston

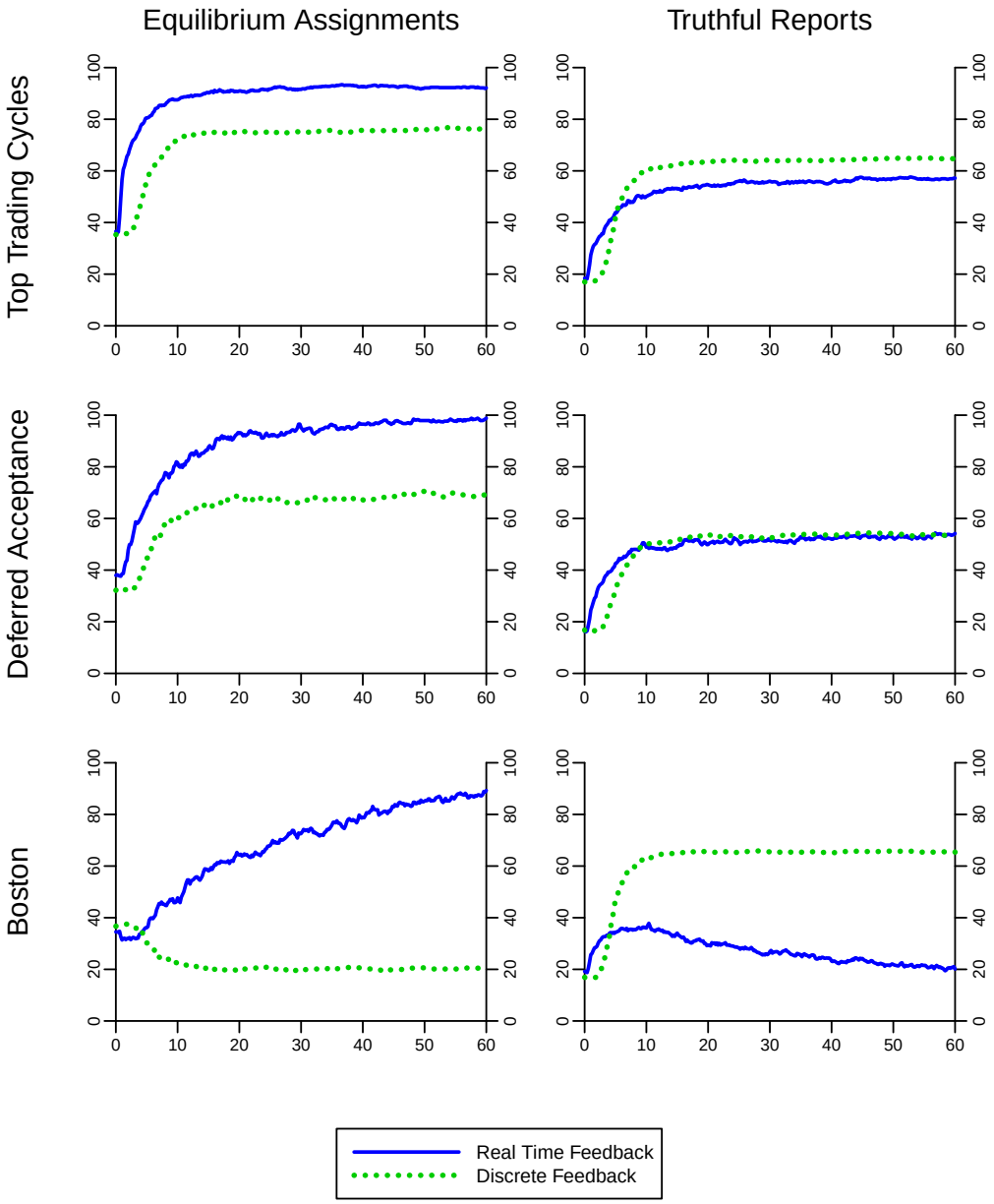

Figure 5: Average behavior within reporting periods

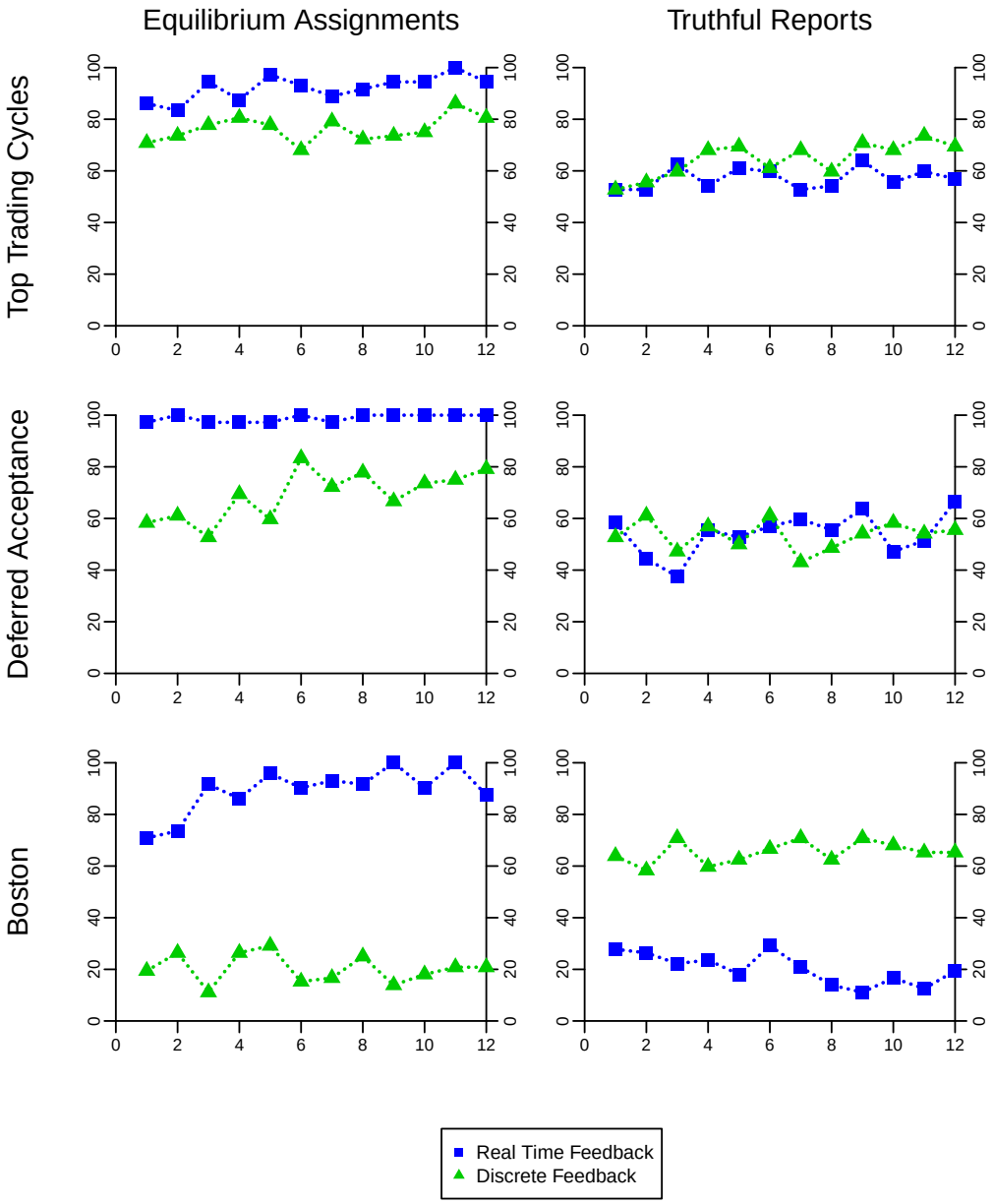

Figure 6: Average behavior across reporting periods

	Feedback		Rank-Sum Test
	Discrete	Real-time	p-value
Top Trading Cycles	83.2	95.5	0.00025
Deferred Acceptance	82.6	99.4	
Boston	58.8	94.1	

Table 4: Best responses by treatment

	Feedback		Rank-Sum Test
	Discrete	Real-time	p-value
Top Trading Cycles	76.3	92.1	0.00025
Deferred Acceptance	69.1	98.8	
Boston	20.3	89.2	

Table 5: Equilibrium assignments by treatment

	Feedback		Rank-Sum Test
	Discrete	Real-time	p-value
Top Trading Cycles	0.500	0.600	0.00025
Deferred Acceptance	0.521	0.954	
Boston	0.562	0.740	

Table 6: Elimination of justified envy by treatment

	Feedback		Rank-Sum Test
	Discrete	Real-time	p-value
Top Trading Cycles	48.7	59.7	0.00025
Deferred Acceptance	18.8	0.6	
Boston	52.4	6.1	

Table 7: Most preferred assignments by treatment

mechanism has no dominant-strategy equilibrium. In the Boston mechanism, adaptive models predict that greater sensitivity to tentative assignments will produce more Nash equilibrium assignments. Consistent with adaptive model predictions, real-time assignment feedback produced more Nash equilibrium assignments in the Boston mechanism. A rank-sum test finds these values to be significantly different across feedback conditions at the 1% level.

Result 1. *Consistent with hypothesis 1, all three mechanisms achieved more equilibrium assignments under real-time feedback than discrete feedback.*

Table 6 provides the percentage of subjects without justified envy. The dominant-strategy equilibrium of the deferred acceptance mechanism and the Nash equilibria of the Boston mechanism fully eliminate justified envy. Consistent with result 1, both of these mechanisms eliminated more justified envy under real-time feedback than discrete feedback. The dominant-strategy equilibrium of the top trading cycles mechanism only eliminates justified envy from two-thirds of the student population. Consistent with result 1, the top trading cycles mechanism was closer to this prediction under real-time feedback than discrete feedback. A rank-sum test finds these values to be significantly different across feedback conditions at the 1% level.

Result 2. *Consistent with hypothesis 2, the deferred acceptance and Boston mechanisms eliminated more justified envy under real-time feedback than discrete feedback.*

Table 7 provides the percentage of subjects who received their most preferred assignment. In this market structure, it is impossible to assign more than two thirds of the student population to their most preferred option. The dominant-strategy Nash equilibrium of the top trading cycles mechanism achieves this upper bound. In contrast, neither the dominant strategy equilibrium of the deferred acceptance mechanism nor the Nash equilibria of the Boston mechanism assign any students to their most preferred school in this market structure. Consistent with result 1, the top trading cycles mechanism assigned more students to their most preferred school under real-time feedback than discrete feedback while the deferred acceptance and Boston mechanisms assigned fewer students to their most preferred school under real-time feedback than discrete

	Feedback		Rank-Sum Test
	Discrete	Real-time	p-value
Top Trading Cycles	0.65	0.57	0.006
Deferred Acceptance	0.54	0.54	
Boston	0.65	0.20	

Table 8: Truthful reporting by treatment

	Feedback		Rank-Sum Test
	Discrete	Real-time	p-value
Top Trading Cycles	0.75	0.81	0.258
Deferred Acceptance	0.62	0.60	
Boston	0.76	0.29	

Table 9: Truthful reporting of top choices by treatment

feedback. A rank-sum test finds these values to be significantly different across feedback conditions at the 1% level.

Result 3. *Consistent with hypothesis 3, top trading cycles assigned more students to their most preferred school under real-time feedback than discrete feedback.*

Every Nash equilibrium of the Boston mechanism fully eliminates justified envy, but it has no dominant strategy Nash equilibrium and none of its equilibria are neutrally stable. Accordingly, adaptive models predict less reliable convergence to equilibrium in the Boston mechanism than the other two mechanisms. Consistent with these predictions, the Boston mechanism eliminated less justified envy than the deferred acceptance mechanism and it assigned fewer students to their most preferred assignment than the top trading cycles mechanism in the real-time feedback treatment. In the discrete feedback treatment, the Boston mechanism eliminated more justified envy than the deferred acceptance mechanism and it assigned more participants to their most preferred option than the top trading cycles mechanism, contrary to equilibrium predictions.

Table 8 provides treatment level averages for truthful reporting by treatment. A preference report is said to be truthful if it accurately reports the agent's entire preference ranking. A rank-sum test finds significantly fewer truthful reports under real-time feedback than discrete feedback at the 1% level. Table 9 provides treatment level averages for the rate at which participants truthfully reported their top choice. In the top trading cycles mechanism, subjects truthfully reported their top choice more often under real-time feedback. In the Boston mechanism, subjects truthfully reported their top choice more often under discrete feedback.

All three mechanisms have Nash equilibria that involve inaccurate preference reports. In strategy proof mechanisms, inaccurate reports often produce the same assignments as truthful reports. In the Boston mechanism, Nash equilibria generally require inaccurate preference reports. Consequently, adaptive models predict that sensitivity to tentative assignments is insufficient to reliably induce truthful reporting. Providing assignment feedback during the preference reporting period makes best responses easier to identify. Subjects might rely less on truthfulness to guide their choice of report when they can easily identify best responses.

In top trading cycles and deferred acceptance, the dominant-strategy equilibrium assigns type 3 students to their least preferred school regardless of their preference report. In the Boston mechanism, all Nash equilibria assign type 3 students to their least preferred school regardless of their preference report. Under real-time feedback, this aspect of the mechanism is easily observed. Subjects who believe their report has little effect on their assignment might tend to select their reports less carefully. Accordingly, type 3 subjects frequently misreported their preferences in mechanisms with real-time feedback. Truthful reporting rates by subject type are provided in the appendix.

Parameters for the adaptive model described in section 3.4 are estimated at the subject level by maximum likelihood. Table 10 provides treatment level averages and hypothesis tests for these parameter estimates. A detailed description of the parameter estimates is provided in the appendix. Rank-sum tests find each parameter to be significantly different across feedback conditions at the 1% level. Each hypothesis test compares 9 discrete feedback sessions with 9 real-time feedback sessions for total of 18 observations. As hypothesized,

Parameter	Feedback Discrete	Feedback Real-Time	Rank-Sum Test p-value
Assignment Sensitivity (α)	0.38	1.79	0.00004
Truthful Tendency (β)	1.73	0.61	0.00004
Behavioral Inertia (γ)	5.40	3.22	0.00004

Table 10: Estimated Behavioral Parameters

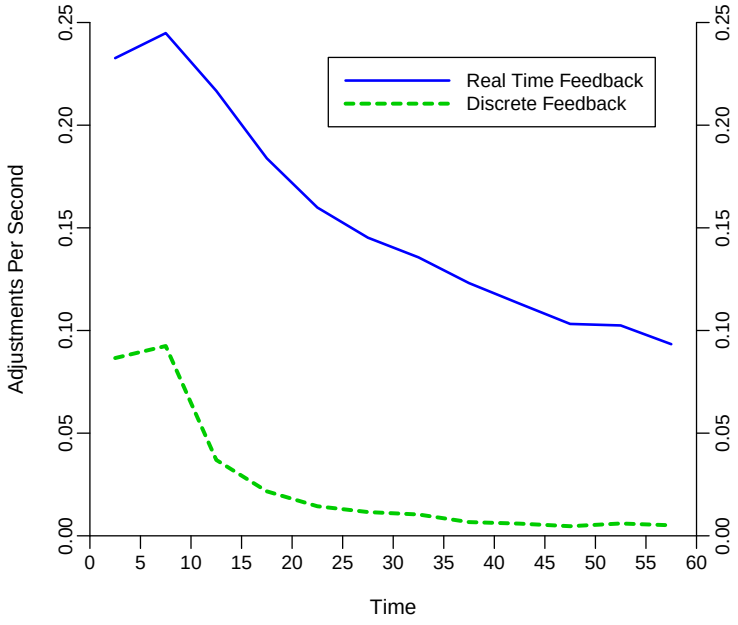

Figure 7: Adjustment rates over time by treatment

subjects exhibited greater sensitivity to tentative assignments under real-time feedback than discrete feedback. In both treatments, subjects tended to select truthful reports more often than inaccurate reports, all else being equal. This tendency was stronger under discrete feedback, but it remained insufficient to reliably produce truthfulness in either treatment. In the absence of feedback, subjects might rely more on truthfulness to guide their choice of reports.

Subjects exhibited stronger behavioral inertia in the discrete feedback treatment than the real-time feedback treatment. Behavioral inertia is the tendency for agents to continue doing what they did in the past. Figure 7 illustrates average adjustment rates in each 5 second interval by treatment. The vertical axis indicates the average number of adjustments per second. The horizontal axis indicates seconds within the reporting period. Consistent with greater behavioral inertia, subjects in the discrete feedback treatment adjusted their reports less frequently than subjects in the real-time feedback treatment. In both treatments, adjustment rates tended to decline over time after the first 10 seconds. In the discrete feedback treatment, the lack of additional information during the reporting period gives subjects little incentive to make further adjustments to their report after making an initial selection. In the real-time feedback treatment, subjects have less incentive to make further adjustments to their reports as the report profile begins to approximate a Nash equilibrium.

7 Conclusion

Conventional student assignment mechanisms only reveal assignments at the end of the reporting period, after all preference reports have been finalized. Adaptive models predict that sensitivity to tentative assignments during the preference reporting period can increase the frequency of equilibrium assignments in three widely employed mechanisms: deferred acceptance, top trading cycles, and the Boston mechanism. To test these predictions, this study compares mechanisms that provide real-time feedback about tentative assignments during the reporting period with mechanisms that only provide feedback at the end of the reporting period.

Real-time assignment feedback consistently produced more equilibrium assignments than discrete feedback in all three mechanisms, suggesting that policy

makers could increase the frequency of equilibrium assignments by providing assignment feedback during the preference reporting period. Consistent with equilibrium predictions, the real-time assignment feedback produced more efficiency in the top trading cycles mechanism and eliminated more justified envy in the deferred acceptance mechanism. No mechanism can guarantee both Pareto efficiency and the elimination of justified envy, so different mechanisms are designed to achieve different policy goals. Because of the fundamental trade-off between Pareto efficiency and the elimination of justified envy, selection between mechanisms ultimately depends on the goals of the policymaker.

Student assignment mechanisms impact the well being of children throughout the world. The New Orleans recovery school district used an algorithm based on the top trading cycles mechanism (Vanacore, 2012). A variation of the deferred acceptance mechanism was employed in New York City (Roth, 2008). The Boston public school system employed the Boston mechanism. Providing feedback about tentative assignments during the preference reporting period could help these mechanisms more reliably achieve policy goals. By increasing the frequency of equilibrium assignments, assignment feedback could help the top trading cycles mechanism achieve more efficiency and help the deferred acceptance mechanism eliminate more justified envy.

In the Boston mechanism, Nash equilibria generally involve inaccurate preference reports. In top trading cycles and deferred acceptance, truthful reporting is weakly dominant but not strictly dominant. All three mechanisms have Nash equilibria involving inaccurate preference reports. Consequently, adaptive models predict that assignment feedback is insufficient to reliably induce truthful reporting. Consistent with adaptive predictions, real-time assignment feedback did not reliably increase the frequency of truthful reporting. Guillen and Hakimov (2018) find that top-down advice increased the rate at which subjects truthful reported their top choices in the top trading cycles mechanism. Future research should investigate the combination of real-time feedback and top down advice in school choice mechanisms.

In practice, school choice mechanisms often involve large numbers of participants and schools, but the present experiment considers a relatively simple market structure with only 24 students and 3 schools. This setting creates an ideal scenario for participants to learn by observing potential assignments from

all possible preference reports which greatly reduces their cognitive load. Field implementations could provide participants with information about all possible reorderings of their top three choices, but it may not be feasible to provide complete feedback about all possible preference reports in settings with a large number of schools. Although promising results were obtained in a lab setting, further research is needed to determine if these results can be extended to more general settings. Additional research is needed to investigate how variations in market structure affect convergence, stability, and equilibrium selection.

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A Appendix

School disctricts frequently allow participants to submit and adjust their preference reports over a period of several days. For example, Polk County Public Schools (2022a) indicates that they gave participants over three weeks to submit their preferences. Their application instructions (2022b) asks participants to "select the schools to which you wish to apply in order of preference" and states that participants can make changes "at any time during the application window." The Pasadena Unified School District (2022c) gave students over two weeks to submit their preferences. Their application instructions (2022d) states that "schools should be listed in order of preference" and that revisions to the list of preferred schools may be made "up until the closing deadline." Dur et al. (2018) notes that the Wake County Public School System gave participants "a two-week window during which they must log into a website and submit their preferences. A student is free to change her ranking as many times as she wishes."

Tables 11, 12, and 13 identify the assignments, reports, and neutral stability of every symmetric Nash equilibrium under deferred acceptance, top trading cycles, and the Boston mechanism in the market structure described by section 3.1. In this market structure, the assignment μ uniquely eliminates justified envy but is Pareto dominated by the assignment λ. Figures 8, 9, and 10 illustrate the observed percentage of equilibrium assignments over time by period and mechanism. Solid lines depict the real-time feedback treatment. Dotted lines depict the discrete feedback treatment. Each graph is labeled with the period it depicts. The horizontal axis indicates time within each reporting period. In strategy-proof mechanisms, the vertical axis indicates the percentage of dominant-strategy equilibrium assignments. In the Boston mechanism, the vertical axis indicates the percentage of Nash equilibrium assignments. Real-time feedback produced more equilibrium assignments in each period of each mechanism. Tables 14-16 provide treatment level averages for truthful reporting by subject type. Table 17 provides subject-level parameter estimates for the model described in section 3.4.

Preference Reports				Neutral
Type 1	Type 2	Type 3	Assignment	Stability
(a,b,c)	(a,b,c)	(a,b,c)	μ	✗
(a,b,c)	(a,b,c)	(a,c,b)	μ	✗
(a,b,c)	(a,b,c)	(b,a,c)	μ	✗
(a,b,c)	(b,a,c)	(a,b,c)	μ	✗
(a,b,c)	(b,a,c)	(a,c,b)	μ	✗
(a,b,c)	(b,a,c)	(b,a,c)	μ	✗
(a,b,c)	(b,a,c)	(b,c,a)	μ	✗
(a,b,c)	(b,a,c)	(c,a,b)	μ	✗
(a,b,c)	(b,a,c)	(c,b,a)	μ	✗
(a,b,c)	(b,c,a)	(a,b,c)	μ	✗
(a,b,c)	(b,c,a)	(a,c,b)	μ	✗
(a,b,c)	(b,c,a)	(b,a,c)	μ	✗
(a,b,c)	(b,c,a)	(b,c,a)	μ	✗
(a,b,c)	(b,c,a)	(c,a,b)	μ	✗
(a,b,c)	(b,c,a)	(c,b,a)	μ	✗
(a,c,b)	(a,b,c)	(a,b,c)	μ	✗
(a,c,b)	(a,b,c)	(a,c,b)	μ	✗
(a,c,b)	(a,b,c)	(b,a,c)	μ	✗
(a,c,b)	(b,a,c)	(a,b,c)	μ	✗
(a,c,b)	(b,a,c)	(a,c,b)	μ	✗
(a,c,b)	(b,a,c)	(b,a,c)	μ	✗
(a,c,b)	(b,a,c)	(b,c,a)	μ	✗
(a,c,b)	(b,a,c)	(c,a,b)	μ	✗
(a,c,b)	(b,a,c)	(c,b,a)	μ	✗
(a,c,b)	(b,a,c)	(a,b,c)	μ	✗
(a,c,b)	(b,a,c)	(a,c,b)	μ	✗
(a,c,b)	(b,a,c)	(b,a,c)	μ	✗
(a,c,b)	(b,a,c)	(b,c,a)	μ	✗
(a,c,b)	(b,a,c)	(c,a,b)	μ	✗
(a,c,b)	(b,a,c)	(c,b,a)	μ	✗
(b,a,c)	(a,b,c)	(a,b,c)	μ	✓
(b,a,c)	(a,b,c)	(a,c,b)	μ	✓
(b,a,c)	(a,b,c)	(b,a,c)	μ	✗
(b,a,c)	(a,b,c)	(b,c,a)	λ	✗
(b,a,c)	(a,b,c)	(c,a,b)	λ	✗
(b,a,c)	(a,b,c)	(c,b,a)	λ	✗
(b,a,c)	(b,a,c)	(a,b,c)	μ	✗
(b,a,c)	(b,a,c)	(a,c,b)	μ	✗
(b,a,c)	(b,a,c)	(b,a,c)	μ	✗
(b,a,c)	(b,c,a)	(a,b,c)	μ	✗
(b,a,c)	(b,c,a)	(a,c,b)	μ	✗
(b,a,c)	(b,c,a)	(b,a,c)	μ	✗

Table 11: Symmetric Nash Equilibria of the Deferred Acceptance Mechanism

Preference Reports				Neutral
Type 1	Type 2	Type 3	Assignment	Stability
(a,b,c)	(b,a,c)	(a,b,c)	μ	✗
(a,b,c)	(b,a,c)	(a,c,b)	μ	✗
(a,b,c)	(b,a,c)	(b,a,c)	μ	✗
(a,b,c)	(b,a,c)	(b,c,a)	μ	✗
(a,b,c)	(b,a,c)	(c,a,b)	μ	✗
(a,b,c)	(b,a,c)	(c,b,a)	μ	✗
(a,b,c)	(b,c,a)	(a,b,c)	μ	✗
(a,b,c)	(b,c,a)	(a,c,b)	μ	✗
(a,b,c)	(b,c,a)	(b,a,c)	μ	✗
(a,b,c)	(b,c,a)	(b,c,a)	μ	✗
(a,b,c)	(b,c,a)	(c,a,b)	μ	✗
(a,b,c)	(b,c,a)	(c,b,a)	μ	✗
(a,c,b)	(b,a,c)	(a,b,c)	μ	✗
(a,c,b)	(b,a,c)	(a,c,b)	μ	✗
(a,c,b)	(b,a,c)	(b,a,c)	μ	✗
(a,c,b)	(b,a,c)	(b,c,a)	μ	✗
(a,c,b)	(b,a,c)	(c,a,b)	μ	✗
(a,c,b)	(b,a,c)	(c,b,a)	μ	✗
(a,c,b)	(b,c,a)	(a,b,c)	μ	✗
(a,c,b)	(b,c,a)	(a,c,b)	μ	✗
(a,c,b)	(b,c,a)	(b,a,c)	μ	✗
(a,c,b)	(b,c,a)	(b,c,a)	μ	✗
(a,c,b)	(b,c,a)	(c,a,b)	μ	✗
(a,c,b)	(b,c,a)	(c,b,a)	μ	✗
(b,a,c)	(a,b,c)	(a,b,c)	λ	✓
(b,a,c)	(a,b,c)	(a,c,b)	λ	✓
(b,a,c)	(a,b,c)	(b,a,c)	λ	✓
(b,a,c)	(a,b,c)	(b,c,a)	λ	✓
(b,a,c)	(a,b,c)	(c,a,b)	λ	✓
(b,a,c)	(a,b,c)	(c,b,a)	λ	✓
(b,a,c)	(a,c,b)	(a,b,c)	λ	✗
(b,a,c)	(a,c,b)	(a,c,b)	λ	✗
(b,a,c)	(a,c,b)	(b,a,c)	λ	✗
(b,a,c)	(a,c,b)	(b,c,a)	λ	✗
(b,a,c)	(a,c,b)	(c,a,b)	λ	✗
(b,a,c)	(a,c,b)	(c,b,a)	λ	✗
(b,c,a)	(a,b,c)	(a,b,c)	λ	✗
(b,c,a)	(a,b,c)	(a,c,b)	λ	✗
(b,c,a)	(a,b,c)	(b,a,c)	λ	✗
(b,c,a)	(a,b,c)	(b,c,a)	λ	✗
(b,c,a)	(a,b,c)	(c,a,b)	λ	✗
(b,c,a)	(a,b,c)	(c,b,a)	λ	✗
(b,c,a)	(a,c,b)	(a,b,c)	λ	✗
(b,c,a)	(a,c,b)	(a,c,b)	λ	✗
(b,c,a)	(a,c,b)	(b,a,c)	λ	✗
(b,c,a)	(a,c,b)	(b,c,a)	λ	✗
(b,c,a)	(a,c,b)	(c,a,b)	λ	✗
(b,c,a)	(a,c,b)	(c,b,a)	λ	✗

Table 12: Symmetric Nash Equilibria of the Top Trading Cycles Mechanism

Preference Reports				Neutral
Type 1	Type 2	Type 3	Assignment	Stability
(a,b,c)	(b,a,c)	(a,b,c)	μ	✗
(a,b,c)	(b,a,c)	(a,c,b)	μ	✗
(a,b,c)	(b,a,c)	(b,a,c)	μ	✗
(a,b,c)	(b,a,c)	(b,c,a)	μ	✗
(a,b,c)	(b,a,c)	(c,a,b)	μ	✗
(a,b,c)	(b,a,c)	(c,b,a)	μ	✗
(a,b,c)	(b,c,a)	(a,b,c)	μ	✗
(a,b,c)	(b,c,a)	(a,c,b)	μ	✗
(a,b,c)	(b,c,a)	(b,a,c)	μ	✗
(a,b,c)	(b,c,a)	(b,c,a)	μ	✗
(a,b,c)	(b,c,a)	(c,a,b)	μ	✗
(a,b,c)	(b,c,a)	(c,b,a)	μ	✗
(a,c,b)	(b,a,c)	(a,b,c)	μ	✗
(a,c,b)	(b,a,c)	(a,c,b)	μ	✗
(a,c,b)	(b,a,c)	(b,a,c)	μ	✗
(a,c,b)	(b,a,c)	(b,c,a)	μ	✗
(a,c,b)	(b,a,c)	(c,a,b)	μ	✗
(a,c,b)	(b,a,c)	(c,b,a)	μ	✗
(a,c,b)	(b,c,a)	(a,b,c)	μ	✗
(a,c,b)	(b,c,a)	(a,c,b)	μ	✗
(a,c,b)	(b,c,a)	(b,a,c)	μ	✗
(a,c,b)	(b,c,a)	(b,c,a)	μ	✗
(a,c,b)	(b,c,a)	(c,a,b)	μ	✗
(a,c,b)	(b,c,a)	(c,b,a)	μ	✗

Table 13: Symmetric Nash Equilibria of the Boston Mechanism

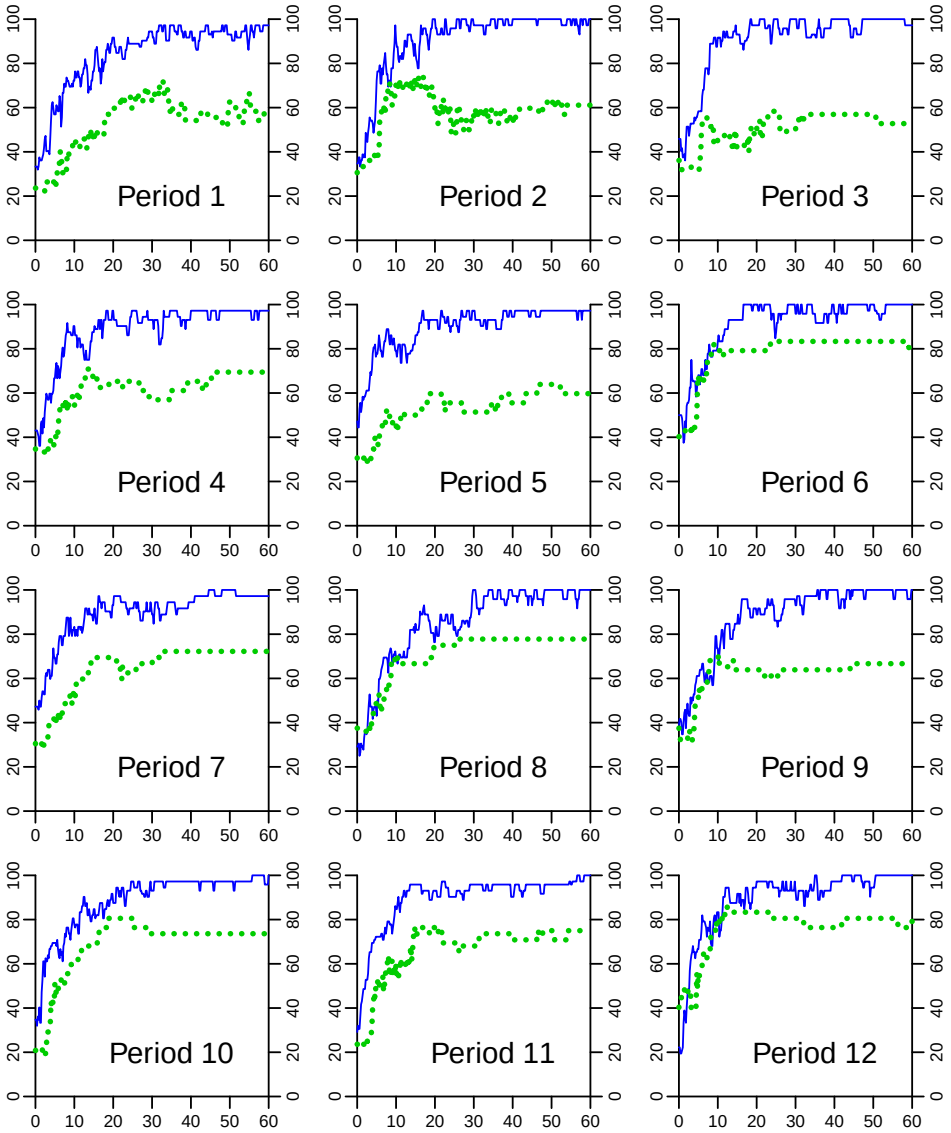

Figure 8: Dominant-strategy equilibrium assignments by period and second in the deferred acceptance mechanism. Solid lines depict the real-time feedback treatment. Dotted lines depict the discrete feedback treatment.

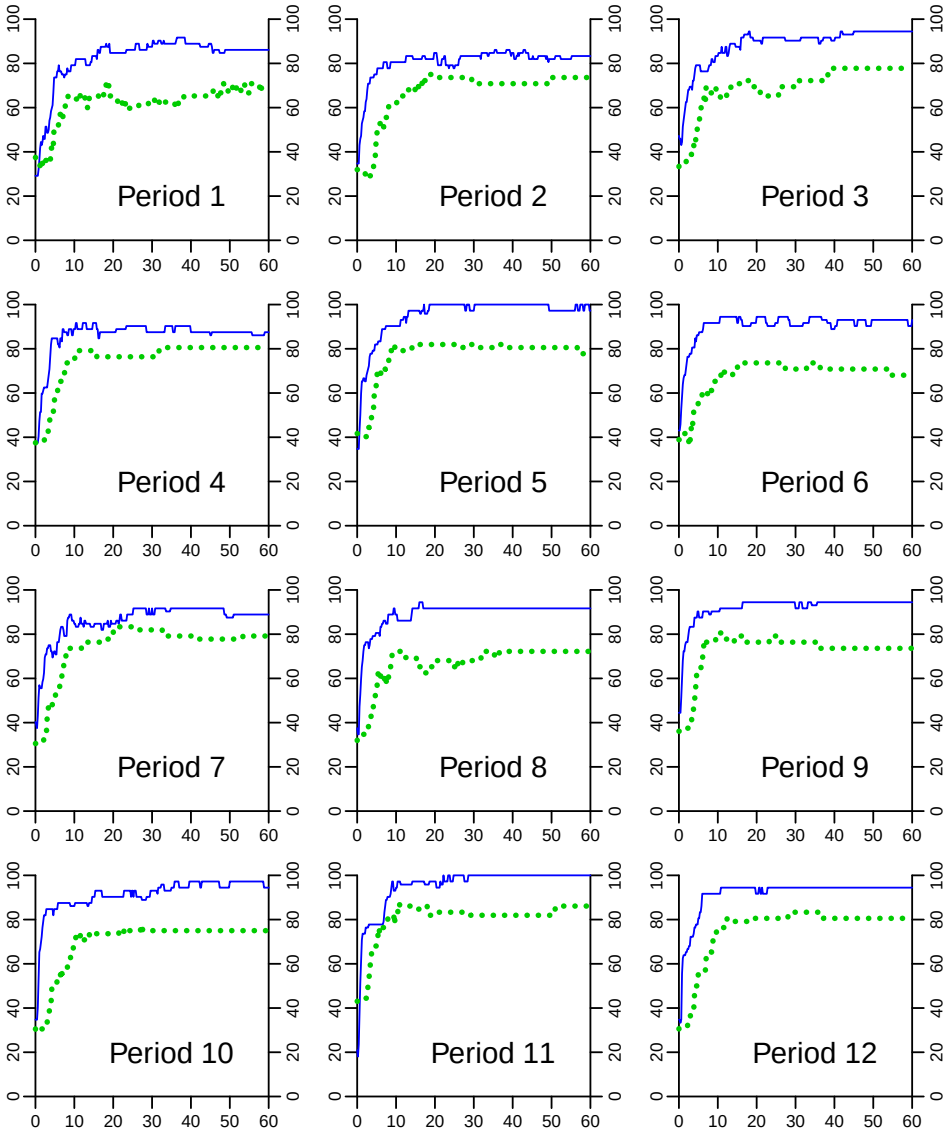

Figure 9: Dominant-strategy equilibrium assignments by period and second in the top trading cycles mechanism. Solid lines depict the real-time feedback treatment. Dotted lines depict the discrete feedback treatment.

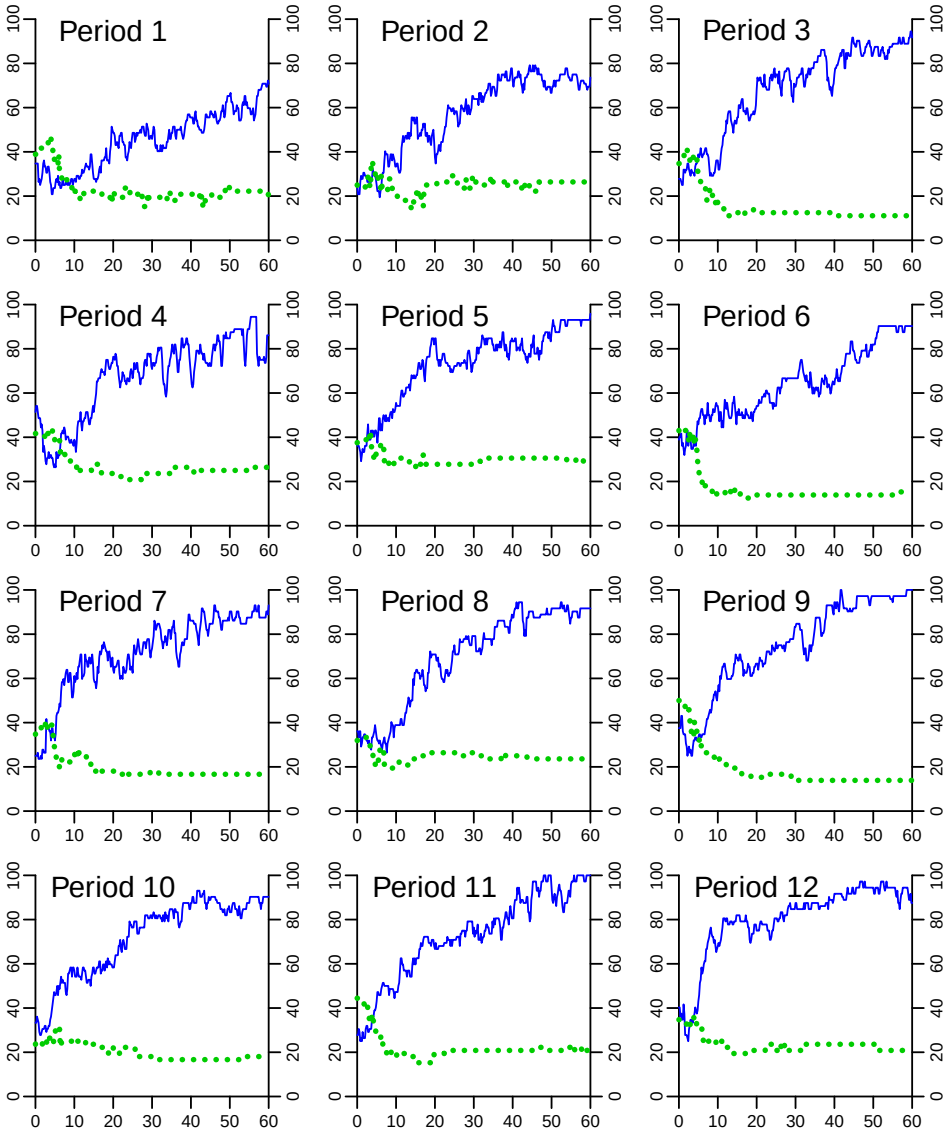

Figure 10: Nash equilibrium assignments by period and second in the Boston mechanism. Solid lines depict the real-time feedback treatment. Dotted lines depict the discrete feedback treatment.

	Feedback	
	Discrete	Real-time
Top Trading Cycles	0.65	0.64
Deferred Acceptance	0.54	0.69
Boston	0.69	0.10

Table 14: Truthful reporting by type 1 subjects

	Feedback	
	Discrete	Real-time
Top Trading Cycles	0.66	0.65
Deferred Acceptance	0.50	0.55
Boston	0.66	0.02

Table 15: Truthful reporting by type 2 subjects

	Feedback	
	Discrete	Real-time
Top Trading Cycles	0.62	0.42
Deferred Acceptance	0.57	0.39
Boston	0.61	0.48

Table 16: Truthful reporting by type 3 subjects

Table 17: Subject Level Estimates

Session	Feedback	Mechanism	Subject	α	β	γ
1	real time	deferred acceptance	1	1.62	0.76	3.90
1	real time	deferred acceptance	2	0.62	1.21	4.22
1	real time	deferred acceptance	3	2.31	0.93	3.89
1	real time	deferred acceptance	4	1.86	0.40	3.33
1	real time	deferred acceptance	5	3.09	1.36	4.77
1	real time	deferred acceptance	6	2.28	0.65	3.36
1	real time	deferred acceptance	7	2.09	1.25	3.99
1	real time	deferred acceptance	8	1.79	0.06	3.40
1	real time	deferred acceptance	9	1.10	0.32	3.54
1	real time	deferred acceptance	10	2.27	0.49	3.01
1	real time	deferred acceptance	11	1.80	0.42	2.42
1	real time	deferred acceptance	12	1.26	2.03	2.69
1	real time	deferred acceptance	13	2.62	0.00	4.21
1	real time	deferred acceptance	14	2.45	0.00	4.40
1	real time	deferred acceptance	15	2.46	1.96	3.86
1	real time	deferred acceptance	16	1.47	1.22	3.58
1	real time	deferred acceptance	17	1.59	0.28	3.16
1	real time	deferred acceptance	18	1.42	0.54	2.79
1	real time	deferred acceptance	19	2.09	0.10	4.52
1	real time	deferred acceptance	20	1.07	0.37	2.28
1	real time	deferred acceptance	21	1.21	2.03	4.65
1	real time	deferred acceptance	22	2.12	0.29	2.82
1	real time	deferred acceptance	23	2.41	0.04	3.18
1	real time	deferred acceptance	24	0.75	0.00	2.85
2	real time	boston	1	1.45	0.84	2.10
2	real time	boston	2	1.20	0.38	3.65
2	real time	boston	3	1.37	0.07	2.36
2	real time	boston	4	1.17	1.52	3.62
2	real time	boston	5	2.69	1.62	1.73
2	real time	boston	6	1.28	0.21	2.47
2	real time	boston	7	0.86	0.00	3.65
2	real time	boston	8	2.49	0.34	3.17
2	real time	boston	9	2.99	0.42	2.89
2	real time	boston	10	0.90	0.62	1.99
2	real time	boston	11	1.85	0.39	2.71
2	real time	boston	12	2.29	1.27	2.54
2	real time	boston	13	1.68	0.32	2.21
2	real time	boston	14	2.18	0.10	3.10

2	real time	boston	15	1.70	0.28	0.84
2	real time	boston	16	2.25	0.82	2.96
2	real time	boston	17	1.86	1.50	1.86
2	real time	boston	18	1.51	0.69	2.99
2	real time	boston	19	0.67	0.31	2.32
2	real time	boston	20	1.28	0.36	2.65
2	real time	boston	21	1.42	0.97	2.49
2	real time	boston	22	1.99	0.40	3.56
2	real time	boston	23	0.93	0.51	2.71
2	real time	boston	24	1.31	0.65	2.24
3	real time	top trading	1	1.52	0.13	2.50
3	real time	top trading	2	1.36	0.15	4.10
3	real time	top trading	3	1.77	0.25	2.79
3	real time	top trading	4	2.15	1.15	4.64
3	real time	top trading	5	6.22	1.90	3.14
3	real time	top trading	6	4.55	0.77	3.91
3	real time	top trading	7	2.38	0.52	4.66
3	real time	top trading	8	5.12	0.00	5.51
3	real time	top trading	9	2.58	0.42	3.01
3	real time	top trading	10	1.09	0.74	3.34
3	real time	top trading	11	1.71	0.00	2.40
3	real time	top trading	12	1.46	0.07	2.88
3	real time	top trading	13	0.22	2.32	4.17
3	real time	top trading	14	4.02	0.03	1.38
3	real time	top trading	15	2.79	0.00	3.81
3	real time	top trading	16	1.94	0.12	4.16
3	real time	top trading	17	0.09	0.00	5.77
3	real time	top trading	18	0.81	0.05	4.29
3	real time	top trading	19	1.76	0.00	3.96
3	real time	top trading	20	0.81	1.45	3.06
3	real time	top trading	21	1.05	1.34	3.73
3	real time	top trading	22	1.07	0.51	4.44
3	real time	top trading	23	3.98	1.48	3.34
3	real time	top trading	24	6.00	0.53	3.23
4	discrete	boston	1	0.20	0.67	5.08
4	discrete	boston	2	0.62	0.00	5.92
4	discrete	boston	3	0.00	1.57	5.24
4	discrete	boston	4	0.02	0.51	16.83
4	discrete	boston	5	0.92	1.58	6.11
4	discrete	boston	6	0.69	4.01	6.82
4	discrete	boston	7	0.00	3.68	5.38

4	discrete	boston	8	0.00	3.49	4.80
4	discrete	boston	9	0.69	0.19	5.68
4	discrete	boston	10	0.41	0.19	5.53
4	discrete	boston	11	0.00	3.36	5.61
4	discrete	boston	12	0.00	0.84	5.67
4	discrete	boston	13	0.00	3.66	5.22
4	discrete	boston	14	0.31	3.87	5.56
4	discrete	boston	15	0.00	1.12	6.09
4	discrete	boston	16	0.03	1.45	5.62
4	discrete	boston	17	0.00	0.00	5.17
4	discrete	boston	18	0.00	5.89	7.13
4	discrete	boston	19	0.23	5.48	6.57
4	discrete	boston	20	0.55	1.92	5.55
4	discrete	boston	21	0.02	0.91	5.38
4	discrete	boston	22	1.05	0.66	4.93
4	discrete	boston	23	0.60	2.36	5.22
4	discrete	boston	24	0.00	3.47	4.84
5	discrete	deferred acceptance	1	0.04	0.00	5.17
5	discrete	deferred acceptance	2	0.07	0.40	5.59
5	discrete	deferred acceptance	3	0.04	1.32	3.60
5	discrete	deferred acceptance	4	0.29	1.13	5.37
5	discrete	deferred acceptance	5	0.22	0.39	3.24
5	discrete	deferred acceptance	6	0.14	1.03	5.06
5	discrete	deferred acceptance	7	0.75	0.81	4.55
5	discrete	deferred acceptance	8	0.00	3.65	5.16
5	discrete	deferred acceptance	9	0.08	2.00	5.40
5	discrete	deferred acceptance	10	0.00	6.07	7.42
5	discrete	deferred acceptance	11	0.29	3.20	4.82
5	discrete	deferred acceptance	12	0.00	2.25	5.67
5	discrete	deferred acceptance	13	0.00	0.00	4.13
5	discrete	deferred acceptance	14	1.21	0.27	6.55
5	discrete	deferred acceptance	15	0.43	0.83	5.30
5	discrete	deferred acceptance	16	0.81	0.65	5.65
5	discrete	deferred acceptance	17	0.75	0.88	3.46
5	discrete	deferred acceptance	18	1.45	1.01	5.47
5	discrete	deferred acceptance	19	0.60	0.42	4.75
5	discrete	deferred acceptance	20	1.00	2.70	4.46
5	discrete	deferred acceptance	21	0.00	1.42	4.57
5	discrete	deferred acceptance	22	0.29	3.11	5.19
5	discrete	deferred acceptance	23	0.15	0.31	4.94
5	discrete	deferred acceptance	24	0.00	1.19	5.45

6	real time	deferred acceptance	1	1.52	0.07	3.29
6	real time	deferred acceptance	2	2.01	0.87	2.48
6	real time	deferred acceptance	3	1.64	1.05	1.99
6	real time	deferred acceptance	4	1.67	0.44	2.75
6	real time	deferred acceptance	5	1.51	1.27	2.27
6	real time	deferred acceptance	6	0.51	1.62	2.84
6	real time	deferred acceptance	7	0.60	2.38	3.53
6	real time	deferred acceptance	8	0.69	1.08	5.13
6	real time	deferred acceptance	9	5.22	0.27	4.86
6	real time	deferred acceptance	10	1.09	1.49	2.08
6	real time	deferred acceptance	11	2.54	0.62	3.23
6	real time	deferred acceptance	12	2.46	0.36	0.00
6	real time	deferred acceptance	13	1.73	0.56	3.11
6	real time	deferred acceptance	14	3.68	0.30	1.48
6	real time	deferred acceptance	15	2.53	0.69	2.67
6	real time	deferred acceptance	16	0.29	1.47	2.61
6	real time	deferred acceptance	17	1.82	0.30	2.41
6	real time	deferred acceptance	18	2.03	0.25	2.27
6	real time	deferred acceptance	19	2.03	0.14	0.27
6	real time	deferred acceptance	20	1.46	0.06	2.25
6	real time	deferred acceptance	21	0.55	0.75	3.86
6	real time	deferred acceptance	22	0.44	1.52	2.45
6	real time	deferred acceptance	23	0.53	0.00	3.95
6	real time	deferred acceptance	24	1.32	0.34	2.46
7	discrete	top trading	1	0.35	3.95	5.26
7	discrete	top trading	2	0.00	3.58	4.67
7	discrete	top trading	3	0.00	1.67	4.06
7	discrete	top trading	4	0.34	1.56	4.45
7	discrete	top trading	5	0.91	3.89	6.56
7	discrete	top trading	6	0.60	2.53	6.28
7	discrete	top trading	7	0.00	3.61	5.95
7	discrete	top trading	8	0.00	0.00	5.78
7	discrete	top trading	9	0.35	3.99	5.09
7	discrete	top trading	10	0.00	0.44	5.53
7	discrete	top trading	11	1.34	1.09	5.46
7	discrete	top trading	12	0.00	1.23	5.16
7	discrete	top trading	13	0.00	0.43	3.79
7	discrete	top trading	14	0.55	0.00	5.43
7	discrete	top trading	15	0.00	1.78	5.46
7	discrete	top trading	16	0.00	5.55	7.37
7	discrete	top trading	17	0.02	3.96	6.42

7	discrete	top trading	18	1.11	0.00	5.20
7	discrete	top trading	19	0.23	0.20	5.16
7	discrete	top trading	20	0.00	3.31	5.58
7	discrete	top trading	21	0.00	1.76	5.43
7	discrete	top trading	22	0.65	0.00	5.33
7	discrete	top trading	23	0.15	0.00	5.22
7	discrete	top trading	24	0.40	0.26	5.09
8	real time	top trading	1	0.76	0.69	2.15
8	real time	top trading	2	1.64	0.57	4.45
8	real time	top trading	3	0.29	3.34	5.46
8	real time	top trading	4	1.10	0.15	3.48
8	real time	top trading	5	0.53	1.84	3.48
8	real time	top trading	6	2.91	1.48	5.57
8	real time	top trading	7	12.13	1.95	2.55
8	real time	top trading	8	4.44	1.13	4.73
8	real time	top trading	9	2.27	0.69	4.42
8	real time	top trading	10	1.97	0.53	4.79
8	real time	top trading	11	1.28	0.93	5.13
8	real time	top trading	12	1.58	0.00	5.13
8	real time	top trading	13	0.70	2.27	3.80
8	real time	top trading	14	1.94	0.02	3.90
8	real time	top trading	15	0.29	0.00	3.73
8	real time	top trading	16	1.63	0.35	5.32
8	real time	top trading	17	6.99	0.25	4.15
8	real time	top trading	18	1.73	0.00	4.45
8	real time	top trading	19	1.29	1.17	3.41
8	real time	top trading	20	3.71	0.10	3.69
8	real time	top trading	21	2.13	0.27	3.20
8	real time	top trading	22	12.23	0.77	3.89
8	real time	top trading	23	1.78	1.22	4.30
8	real time	top trading	24	0.38	2.65	3.50
9	real time	boston	1	1.92	1.38	1.28
9	real time	boston	2	1.87	0.50	2.27
9	real time	boston	3	1.02	0.55	2.36
9	real time	boston	4	2.18	0.30	3.11
9	real time	boston	5	2.47	1.24	1.87
9	real time	boston	6	0.98	0.73	3.43
9	real time	boston	7	1.38	0.24	2.44
9	real time	boston	8	1.15	0.24	3.23
9	real time	boston	9	1.19	0.00	4.48
9	real time	boston	10	1.42	0.00	3.46

9	real time	boston	11	1.58	0.17	2.08
9	real time	boston	12	0.35	0.22	1.76
9	real time	boston	13	2.80	0.81	3.72
9	real time	boston	14	1.73	0.00	5.18
9	real time	boston	15	1.56	0.99	2.64
9	real time	boston	16	1.87	0.39	2.35
9	real time	boston	17	1.83	0.00	1.90
9	real time	boston	18	1.51	0.68	2.07
9	real time	boston	19	1.28	0.20	2.51
9	real time	boston	20	1.40	0.00	3.16
9	real time	boston	21	1.27	0.20	2.92
9	real time	boston	22	1.13	0.47	2.19
9	real time	boston	23	1.26	0.16	2.09
9	real time	boston	24	1.26	0.00	2.86
10	discrete	boston	1	0.61	0.00	6.29
10	discrete	boston	2	0.22	0.51	5.67
10	discrete	boston	3	0.65	0.78	5.60
10	discrete	boston	4	0.03	2.04	5.62
10	discrete	boston	5	0.00	1.92	4.94
10	discrete	boston	6	0.64	1.90	5.67
10	discrete	boston	7	0.00	6.10	7.62
10	discrete	boston	8	0.63	0.64	4.38
10	discrete	boston	9	0.01	3.58	4.65
10	discrete	boston	10	0.00	1.37	5.60
10	discrete	boston	11	0.00	4.01	5.83
10	discrete	boston	12	0.00	3.95	4.84
10	discrete	boston	13	0.13	3.55	5.13
10	discrete	boston	14	1.33	0.00	5.78
10	discrete	boston	15	0.05	0.31	5.18
10	discrete	boston	16	0.56	0.00	5.54
10	discrete	boston	17	0.38	0.95	5.53
10	discrete	boston	18	0.37	1.24	5.54
10	discrete	boston	19	0.00	2.60	5.63
10	discrete	boston	20	0.32	1.98	5.09
10	discrete	boston	21	0.03	1.40	5.53
10	discrete	boston	22	0.57	2.29	5.09
10	discrete	boston	23	0.00	4.18	5.47
10	discrete	boston	24	0.00	3.30	4.98
11	discrete	top trading	1	0.53	2.65	5.33
11	discrete	top trading	2	0.10	0.12	5.16
11	discrete	top trading	3	0.10	3.52	5.11

11	discrete	top trading	4	0.35	1.59	6.05
11	discrete	top trading	5	0.00	3.00	4.81
11	discrete	top trading	6	0.00	6.03	7.27
11	discrete	top trading	7	0.48	0.13	5.06
11	discrete	top trading	8	0.58	1.57	4.98
11	discrete	top trading	9	0.96	0.16	5.55
11	discrete	top trading	10	0.10	1.95	5.75
11	discrete	top trading	11	1.23	0.73	5.14
11	discrete	top trading	12	0.64	1.38	4.86
11	discrete	top trading	13	0.64	0.24	5.07
11	discrete	top trading	14	0.47	1.37	4.38
11	discrete	top trading	15	0.28	2.80	4.61
11	discrete	top trading	16	0.00	6.25	7.51
11	discrete	top trading	17	0.57	0.60	5.42
11	discrete	top trading	18	0.19	5.58	6.85
11	discrete	top trading	19	0.60	1.52	5.57
11	discrete	top trading	20	1.38	1.84	6.45
11	discrete	top trading	21	0.31	2.63	5.73
11	discrete	top trading	22	0.16	2.41	4.77
11	discrete	top trading	23	0.35	1.55	5.63
11	discrete	top trading	24	0.00	5.39	7.25
12	discrete	deferred acceptance	1	0.74	0.90	5.29
12	discrete	deferred acceptance	2	1.33	0.53	5.02
12	discrete	deferred acceptance	3	0.39	1.38	5.05
12	discrete	deferred acceptance	4	0.40	1.27	4.91
12	discrete	deferred acceptance	5	0.17	0.02	5.19
12	discrete	deferred acceptance	6	0.00	0.00	5.46
12	discrete	deferred acceptance	7	1.12	0.81	5.25
12	discrete	deferred acceptance	8	0.00	5.58	6.99
12	discrete	deferred acceptance	9	0.22	0.00	5.52
12	discrete	deferred acceptance	10	1.05	0.78	5.66
12	discrete	deferred acceptance	11	0.71	1.80	5.55
12	discrete	deferred acceptance	12	0.95	3.16	6.05
12	discrete	deferred acceptance	13	0.74	0.00	5.62
12	discrete	deferred acceptance	14	1.02	0.00	5.50
12	discrete	deferred acceptance	15	0.00	0.00	5.69
12	discrete	deferred acceptance	16	0.00	2.61	5.71
12	discrete	deferred acceptance	17	0.00	0.00	5.46
12	discrete	deferred acceptance	18	0.49	0.00	6.05
12	discrete	deferred acceptance	19	0.28	0.02	4.90
12	discrete	deferred acceptance	20	0.76	1.75	5.23

12	discrete	deferred acceptance	21	1.56	0.48	5.24
12	discrete	deferred acceptance	22	0.99	0.00	5.73
12	discrete	deferred acceptance	23	1.10	0.86	5.08
12	discrete	deferred acceptance	24	0.95	0.59	5.02
13	discrete	boston	1	0.11	2.97	4.96
13	discrete	boston	2	0.00	2.87	5.25
13	discrete	boston	3	0.61	2.04	6.08
13	discrete	boston	4	0.92	0.32	6.00
13	discrete	boston	5	0.46	0.00	4.87
13	discrete	boston	6	0.43	1.73	3.25
13	discrete	boston	7	0.00	1.98	5.84
13	discrete	boston	8	0.00	4.36	5.43
13	discrete	boston	9	0.00	4.15	5.78
13	discrete	boston	10	0.00	0.37	4.47
13	discrete	boston	11	0.50	0.59	5.74
13	discrete	boston	12	0.35	0.00	5.53
13	discrete	boston	13	0.37	0.04	5.17
13	discrete	boston	14	0.00	3.60	6.13
13	discrete	boston	15	0.13	1.60	4.39
13	discrete	boston	16	0.00	3.70	6.37
13	discrete	boston	17	0.09	1.95	5.40
13	discrete	boston	18	0.00	2.87	2.94
13	discrete	boston	19	0.31	1.39	3.48
13	discrete	boston	20	0.00	0.00	3.99
13	discrete	boston	21	0.00	3.19	5.39
13	discrete	boston	22	0.87	2.74	5.89
13	discrete	boston	23	1.62	0.13	5.52
13	discrete	boston	24	0.22	2.91	4.13
14	real time	boston	1	0.87	0.42	1.84
14	real time	boston	2	0.49	0.44	2.22
14	real time	boston	3	0.36	0.00	4.31
14	real time	boston	4	0.99	0.00	3.27
14	real time	boston	5	1.68	0.15	1.65
14	real time	boston	6	0.59	0.00	2.76
14	real time	boston	7	1.67	0.96	3.10
14	real time	boston	8	1.85	0.40	5.57
14	real time	boston	9	0.94	0.58	2.76
14	real time	boston	10	1.63	0.58	3.60
14	real time	boston	11	1.40	0.00	2.53
14	real time	boston	12	1.69	0.21	4.06
14	real time	boston	13	1.14	1.01	3.82

14	real time	boston	14	1.89	0.68	1.51
14	real time	boston	15	1.60	0.44	3.63
14	real time	boston	16	2.77	0.24	2.95
14	real time	boston	17	1.99	1.06	3.62
14	real time	boston	18	1.35	0.82	3.84
14	real time	boston	19	1.38	0.00	5.07
14	real time	boston	20	3.43	0.00	5.42
14	real time	boston	21	1.22	0.00	1.20
14	real time	boston	22	0.83	0.27	1.68
14	real time	boston	23	1.56	0.92	2.81
14	real time	boston	24	0.69	0.67	3.62
15	real time	deferred acceptance	1	1.37	1.42	3.13
15	real time	deferred acceptance	2	0.61	0.08	2.97
15	real time	deferred acceptance	3	0.89	0.31	2.35
15	real time	deferred acceptance	4	0.95	0.00	3.77
15	real time	deferred acceptance	5	0.52	0.51	1.82
15	real time	deferred acceptance	6	0.93	0.01	3.36
15	real time	deferred acceptance	7	0.34	0.81	2.34
15	real time	deferred acceptance	8	2.22	1.00	3.64
15	real time	deferred acceptance	9	1.36	0.00	3.41
15	real time	deferred acceptance	10	0.88	0.52	4.06
15	real time	deferred acceptance	11	0.82	0.54	2.48
15	real time	deferred acceptance	12	0.53	1.03	3.24
15	real time	deferred acceptance	13	0.77	0.23	2.89
15	real time	deferred acceptance	14	1.59	0.41	3.14
15	real time	deferred acceptance	15	0.55	0.79	2.82
15	real time	deferred acceptance	16	0.51	0.81	3.31
15	real time	deferred acceptance	17	1.51	0.68	4.21
15	real time	deferred acceptance	18	0.84	1.73	3.25
15	real time	deferred acceptance	19	0.33	0.16	2.77
15	real time	deferred acceptance	20	0.91	0.57	3.38
15	real time	deferred acceptance	21	1.56	0.36	2.85
15	real time	deferred acceptance	22	0.74	0.98	4.01
15	real time	deferred acceptance	23	1.27	0.66	2.69
15	real time	deferred acceptance	24	1.71	0.25	3.51
16	real time	top trading	1	1.29	1.35	1.86
16	real time	top trading	2	1.07	0.00	3.57
16	real time	top trading	3	2.21	0.11	2.71
16	real time	top trading	4	0.46	0.56	3.45
16	real time	top trading	5	2.34	0.15	2.82
16	real time	top trading	6	0.65	0.54	3.34

16	real time	top trading	7	3.57	0.14	2.24
16	real time	top trading	8	0.53	1.56	5.50
16	real time	top trading	9	2.05	0.89	2.39
16	real time	top trading	10	2.64	1.19	2.07
16	real time	top trading	11	2.24	0.29	2.53
16	real time	top trading	12	0.78	0.12	3.92
16	real time	top trading	13	0.56	1.66	4.27
16	real time	top trading	14	1.64	0.20	3.02
16	real time	top trading	15	2.68	0.00	3.63
16	real time	top trading	16	0.84	0.00	3.03
16	real time	top trading	17	2.52	0.04	2.98
16	real time	top trading	18	1.81	0.50	3.36
16	real time	top trading	19	0.79	0.24	3.21
16	real time	top trading	20	0.36	0.00	3.73
16	real time	top trading	21	12.05	0.94	3.70
16	real time	top trading	22	0.87	0.07	4.68
16	real time	top trading	23	1.80	0.70	4.01
16	real time	top trading	24	1.47	1.45	3.47
17	discrete	deferred acceptance	1	0.00	0.63	5.20
17	discrete	deferred acceptance	2	0.00	5.80	7.63
17	discrete	deferred acceptance	3	0.00	0.56	4.96
17	discrete	deferred acceptance	4	0.50	0.04	4.96
17	discrete	deferred acceptance	5	1.71	1.38	5.81
17	discrete	deferred acceptance	6	0.00	3.18	4.50
17	discrete	deferred acceptance	7	0.07	0.55	5.17
17	discrete	deferred acceptance	8	0.93	1.36	4.78
17	discrete	deferred acceptance	9	0.00	0.31	4.95
17	discrete	deferred acceptance	10	0.53	0.95	5.22
17	discrete	deferred acceptance	11	0.00	4.36	5.43
17	discrete	deferred acceptance	12	1.15	1.10	5.33
17	discrete	deferred acceptance	13	0.21	0.00	4.34
17	discrete	deferred acceptance	14	0.34	0.20	4.89
17	discrete	deferred acceptance	15	0.19	0.00	4.36
17	discrete	deferred acceptance	16	0.79	1.83	4.34
17	discrete	deferred acceptance	17	0.05	3.74	4.87
17	discrete	deferred acceptance	18	0.00	1.99	3.96
17	discrete	deferred acceptance	19	1.00	1.14	5.33
17	discrete	deferred acceptance	20	0.00	1.47	4.35
17	discrete	deferred acceptance	21	2.04	0.00	7.35
17	discrete	deferred acceptance	22	0.25	2.55	5.60
17	discrete	deferred acceptance	23	0.53	1.33	5.20

17	discrete	deferred acceptance	24	0.00	0.88	2.86
18	discrete	top trading	1	0.00	0.00	4.05
18	discrete	top trading	2	0.03	2.22	5.95
18	discrete	top trading	3	0.16	1.19	5.29
18	discrete	top trading	4	0.00	4.10	5.64
18	discrete	top trading	5	0.52	0.99	5.35
18	discrete	top trading	6	1.32	0.95	5.68
18	discrete	top trading	7	0.06	5.18	7.26
18	discrete	top trading	8	0.61	1.47	5.03
18	discrete	top trading	9	0.87	0.89	5.75
18	discrete	top trading	10	1.74	0.00	5.88
18	discrete	top trading	11	1.13	1.82	4.97
18	discrete	top trading	12	0.00	1.35	4.47
18	discrete	top trading	13	0.24	0.30	5.48
18	discrete	top trading	14	0.32	1.87	5.06
18	discrete	top trading	15	0.63	0.20	5.81
18	discrete	top trading	16	0.77	0.00	5.33
18	discrete	top trading	17	0.00	3.24	5.00
18	discrete	top trading	18	0.00	3.33	5.67
18	discrete	top trading	19	0.94	1.30	5.38
18	discrete	top trading	20	0.00	0.68	5.03
18	discrete	top trading	21	0.10	1.87	5.17
18	discrete	top trading	22	1.00	0.66	5.13
18	discrete	top trading	23	0.75	0.65	5.40
18	discrete	top trading	24	0.00	2.47	5.02