

Continuous Assignment Feedback in School Choice Mechanisms*

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Abstract

Conventional school choice mechanisms reveal assignments to participants only after all preference reports have been finalized. This paper experimentally investigates novel implementations that provide assignment feedback throughout the preference reporting period. Adaptive models predict that such feedback will promote rational preference revelation by providing increased opportunity for learning and adjustment. To test this hypothesis, the experiment compares conventional and novel implementations of three widely employed mechanisms. In line with adaptive predictions, equilibrium assignments were achieved significantly more often under novel implementations. These results suggest that assignment feedback during the preference reporting period can improve the performance of school choice mechanisms.

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1 Introduction

Children in the United States are traditionally assigned to public schools based on where they live, but a growing number of school districts allow parents to indicate their preferences over schools. As each school can support only a limited number of students, it is often infeasible to place every student at her most preferred school. To resolve these shortages, policy makers employ student assignment mechanisms that assign students to schools based on reported student preferences and legally determined student priorities. Conventional implementations of these mechanisms only reveal assignments to participants at the end of the reporting period, after all preference reports have been finalized. This paper experimentally investigates novel implementations that provide participants with assignment feedback throughout the preference reporting period.

Some school choice mechanisms give participants an incentive to strategically misreport¹ their preferences. Misreported preferences can distort assignments and prevent mechanisms from reliably achieving policy goals. To encourage truthful preference reports, economists often recommend strategy-proof assignment mechanisms where participants never have an incentive to misreport their preferences. Yet recent studies² find that even strategy-proof mechanisms fail to reliably induce truthful preference revelation from boundedly rational participants. Adaptive models predict that the provision of assignment feedback throughout the preference reporting period can promote rational preference reporting by providing participants with increased opportunity for learning and adjustment. In contrast, such feedback has no effect on the equilibrium predictions.

To test these hypotheses, the experiment compares conventional feedback implementations with novel continuous assignment feedback implementations of three widely employed school choice mechanisms: the deferred acceptance

¹ Some parent groups have even explicitly recommended particular misreporting strategies. See Abdulkadiroglu et al. (2006) for more details. ² For example, Chen and Sönmez (2006) find that subjects misrepresent their preferences 50% of the time under a top trading cycles mechanism. Similarly, Pais and Pintér (2008) find that subjects misrepresent their preferences 33% of the time under a full information deferred acceptance mechanism.

mechanism, the top trading cycles mechanism, and the Boston mechanism. Under all three mechanisms, subjects were significantly more likely to achieve equilibrium assignments if they received continuous assignment feedback throughout the preference reporting period. In the top trading cycles mechanism, subjects were significantly more likely to obtain their most preferred school if they received continuous assignment feedback. In the deferred acceptance mechanism and the Boston mechanism, subjects were significantly less likely to exhibit justified envy if they received continuous assignment feedback.

Truth telling is a weakly dominant strategy in strategy proof mechanisms. Truthful reports are always optimal, but optimal reports are not always truthful. In general, several distinct preference report profiles may yield the same assignment profile. Hence participants can learn to submit optimal preference reports without learning to submit truthful preference reports. Consequently, continuous feedback implementations of the strategy proof mechanisms achieved significantly more equilibrium assignments, but they did not exhibit significantly more truthful preference reports.

Computational advances have largely eliminated technical barriers to the provision of on-demand assignment feedback throughout the preference reporting period. Many school districts already provide online computerized web interfaces for on-demand preference reporting. Limited forms of feedback have already been employed by some school districts. Feedback regarding the first choices of other participants was provided by the Wake County Public School System (Dur et al., 2015). Inner Mongolia provided feedback in a dynamic queuing mechanism where subjects only report their first choices (Gong and Liang, 2016). It is now computationally feasible to provide participants with on-demand information regarding their school assignment throughout the preference reporting period in strategy-proof preference reporting mechanisms. The experimental results suggest that such feedback may help school choice mechanisms to more reliably achieve policy goals.

The remainder of this paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the mechanisms and environments under consideration. Section 4 presents the experimental design. Section 5 covers the hypothesis. Section 6 presents the results and Section 7 concludes.

2 Related Literature

This study connects two distinct strands of literature: mechanism design and continuous-time experiments. The mechanism design literature provides an axiomatic analysis of rational preference revelation behavior under school choice mechanisms. The present paper investigates three such mechanisms: the deferred acceptance mechanism, the top trading cycles mechanism, and the Boston mechanism. Gale and Shapley (1962) describe the deferred acceptance mechanism, Shapley and Scarf (1974) describe the top trading cycles mechanism, and Ergin and Sönmez (2006) provide a powerful characterization of the Nash equilibria of the Boston mechanism. Abdulkadiroglu and Sönmez (2003) describe the school choice problem and discuss the fundamental tradeoff between Pareto efficiency and the elimination of justified envy.

Previous experimental studies, such as Chen and Sönmez (2006), conducted school choice mechanisms in discrete periods to investigate static one-shot mechanisms. Continuous-time experimental studies have investigated dynamic behavior in a variety of strategic settings. Stephenson (2019) experimentally investigates continuous-time attacker defender games. Cason et al. (2013) experimentally investigates continuous-time rock-paper-scissors games. Oprea et al. (2011) experimentally investigates continuous-time Hawk-Dove games. All three studies provide subjects with continuous feedback and allow subjects to adjust their strategies at will. The present paper employs continuous-time experimental methodology to investigate preference reporting behavior in novel implementations school choice mechanisms.

3 Theory

School choice mechanisms face a fundamental tradeoff between Pareto efficiency and the elimination of justified envy. Subsection 3.1 provides a simple illustration³ of this tradeoff. Subsection 3.2 describes the school choice mechanisms under consideration and their respective equilibria. Subsection 3.3

³ A similar environment with only one student of each type is discussed by Abdulkadiroglu and Sönmez (2003) and Roth (1982).

considers adaptive models and their theoretical predictions.

3.1 The School Choice Environment

Consider a school choice environment where each of three schools can accept up to n students and each student can be assigned to only one school. Each student has strict preferences over schools and each school has a strict priority ranking over students. There are three types of students and n students of each type. Student preferences over schools are given by

Student Type	1	2	3
	<i>b</i>	<i>a</i>	<i>a</i>
Preference	<i>a</i>	<i>b</i>	<i>b</i>
	<i>c</i>	<i>c</i>	<i>c</i>

Here higher vertical position indicates a higher preference ranking. Type 1 students prefer school b over school a and school a over school c . Similarly, school priority rankings over students satisfy

School	<i>a</i>	<i>b</i>	<i>c</i>
	1	2	2
Priority	3	1	1
	2	3	3

Here school a prefers type 1 students over type 3 students and type 3 students over type 2 students. A student x is said to have justified envy towards student y if student x prefers the school assigned to y and student x is also ranked higher than y at this school. If no student has justified envy under a particular assignment of students to schools we say that this assignment eliminates justified envy. In general, several distinct assignments may eliminate justified envy in a given school choice environment. In this particular environment, the only assignment that eliminates justified envy is given by

$$\mu = \begin{pmatrix} 1 & 2 & 3 \\ a & b & c \end{pmatrix}$$

Here all type 1 students are assigned to school a , all type 2 students are assigned to school b , and all type 3 students are assigned to school c . Yet μ is Pareto dominated by the assignment

$$\lambda = \begin{pmatrix} 1 & 2 & 3 \\ b & a & c \end{pmatrix}$$

Here all type 1 students are assigned to school b , all type 2 students are assigned to school a , and all type 3 students are assigned to school c . The assignment λ Pareto dominates the assignment μ because type 1 students and type 2 student both prefer the schools they receive under λ to the schools they receive under μ and type 3 students receive the same school under both λ and μ .

The assignment λ is Pareto optimal but fails to fully eliminate justified envy because it gives type 3 students justified envy towards type 2 students. Type 3 students would prefer school a over school c , and school a ranks type 3 students higher than type 2 students. Since μ uniquely eliminates justified envy and is Pareto dominated by λ , no Pareto optimal assignment can eliminate justified envy in this environment. In general, no student assignment mechanism can guarantee both Pareto optimality and the elimination justified envy.

3.2 Student Assignment Mechanisms

This paper considers three widely employed assignment mechanisms: the Boston mechanism, the top trading cycles mechanism, and the student optimal deferred acceptance mechanism. The deferred acceptance mechanism and the top trading cycles mechanism are both strategy proof. The dominant strategy Nash equilibrium of the deferred acceptance mechanisms always eliminates justified envy. The dominant strategy Nash equilibrium of the top-trading cycles mechanism always achieves Pareto efficiency. The Boston mechanism is manipulable, but every Nash equilibrium of the Boston mechanism eliminates justified envy.

3.2.1 The Boston Mechanism

Under the Boston mechanism, each student initially applies to her top choice of schools according to her reported preferences. Each school accepts applicants in priority order until it runs out of seats. The remaining students apply to their second choice of schools according to their reported preferences. Again, each school accepts students in priority order until it runs out of seats. This process repeats until every student is assigned to a school. If students report truthfully, the Boston mechanism will select a Pareto optimal assignment. However, students can often benefit by misreporting their preferences. Ergin and Sönmez (2006) show that the set of Nash equilibrium assignments for the Boston Mechanism coincide exactly with the set of assignments that eliminate justified envy under the true preferences.

3.2.2 The Deferred Acceptance Mechanism

Under the student optimal deferred acceptance mechanism, each student initially applies to her top choice of schools according to her reported preferences. Each school tentatively accepts applicants in priority order until it runs out of seats. The remaining applications are rejected. Students whose applications were rejected then apply to their next highest choice of schools. Each school then considers its new applicants alongside those it has already tentatively accepted. It tentatively accepts its top priority students among this group until it run out of seats and rejects the remaining students. This process repeats until every student is assigned to a school.

3.2.3 The Top Trading Cycles Mechanism

The top trading cycles mechanism constructs a directed graph based the priorities and reported preferences. Each school points to it's highest priority student and each student points to her most preferred school according to her reported preferences. Since there are a finite number of schools and students, the directed graph will have at least one cycle. Students who are part of a cycle are assigned to the school they point at. Each of the remaining students point to their most preferred school according to their reported preferences

among those schools that still have open seats. Each school points to their highest priority student among those students that remain unassigned. Students who are part of a cycle are assigned to the school they point to. This process repeats until every student is assigned to a school.

3.3 Adaptive Dynamics

The best response dynamic considered by Gilboa and Matsui (1991), describes agents who asynchronously switch to their myopic best responses. It is closely related to the fictitious play dynamic discussed by Brown (1951). Under the best response dynamic, if the currently selected report profile is given by $r \in R$ then an agent i will switch from her current report $r_i \in R_i$ to an alternate report $q_i \in R_i$ with probability $P_i(q_i|r)$ given by

$$P_i(q_i|r) = \frac{f(q_i|r)}{\sum_{x_i \in R_i} f(x_i|r)} \quad \text{where}$$

$$f(x_i|r) = \begin{cases} 1 & \text{if } x_i \in \operatorname{argmax}_{y_i \in R_i} \pi_i(y_i, r_{-i}) \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 and Figure 2 depict predictions of the best response dynamic. In both figures, the horizontal axis denotes time over the course of a reporting period. In Figure 1 the vertical axis denotes the percentage of participants receiving their equilibrium assignment. In Figure 2 the vertical axis depicts the percentage of participants sending truthful preference reports. Each line in each figure depicts the mean path of a particular mechanism under the best response dynamic in the environment described by Subsection 3.1.

The best response dynamic predicts that providing assignment feedback throughout the preference reporting period will help these mechanisms achieve equilibrium assignments. In contrast, such feedback has no effect on the Nash equilibrium predictions as assignments remain exclusively determined by the preference reports selected at the end of the reporting period.

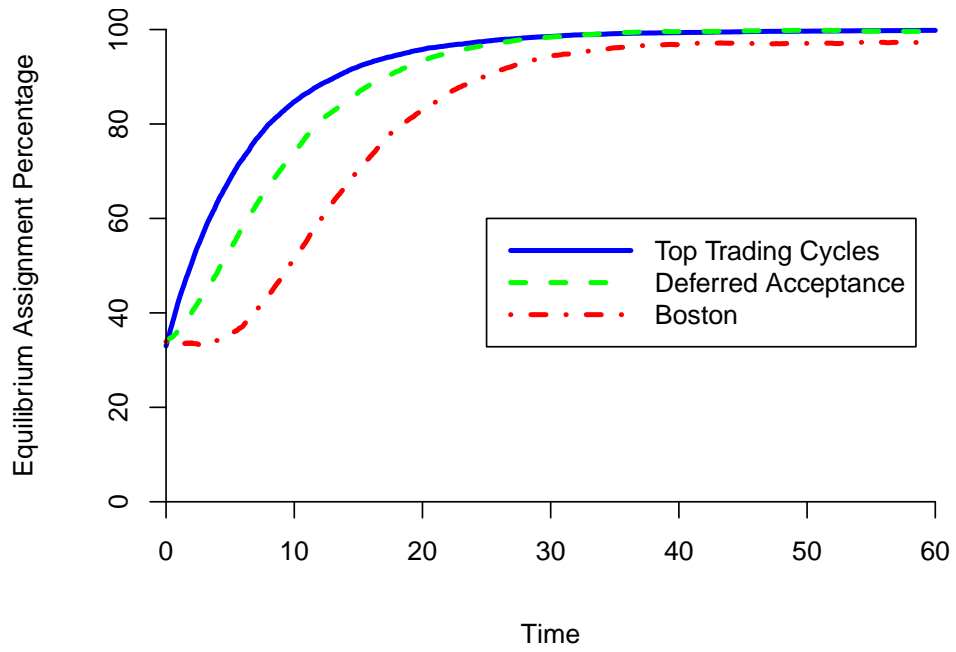


Figure 1: Equilibrium Assignment Percentages under the Best Response Dynamic

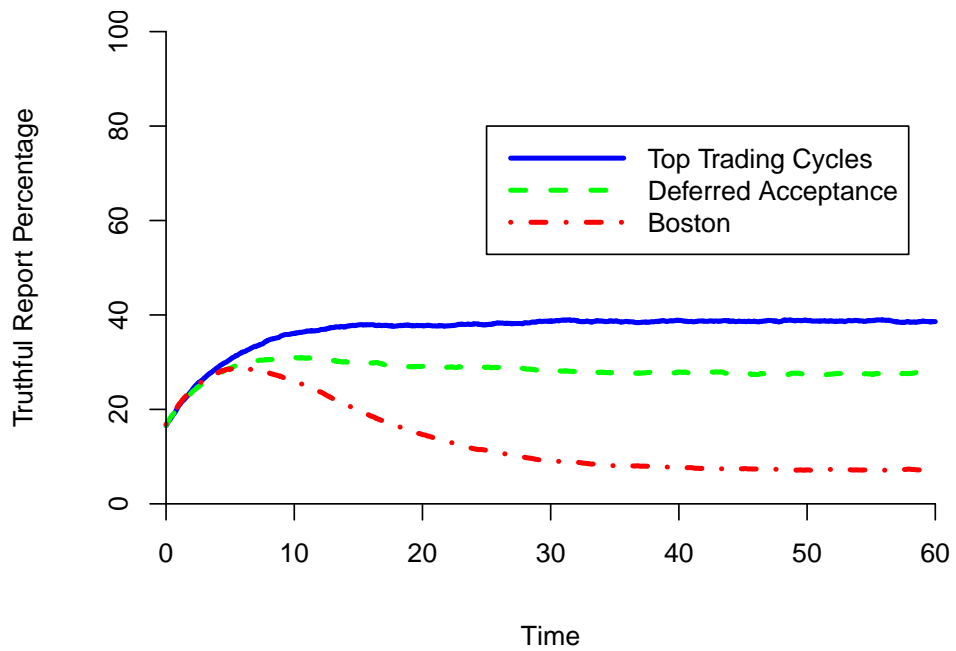


Figure 2: Truthful Report Percentages under the Best Response Dynamic

In strategy proof mechanisms, truthful preference reports are always optimal, but optimal preference reports are not always truthful. Distinct report profiles can often yield identical assignment profiles. In the deferred acceptance mechanism, if type 1 and type 2 students report truthfully then type 3 students will be assigned to school c regardless of their preference report, so they have no incentive to report truthfully. Similarly, in the top trading cycles mechanism, if type 2 and type 3 students report truthfully then type 1 students will be assigned to school b so long as they list it as their first choice, regardless of how they rank school a and school c . In this case, type 1 students have no incentive to select the truthful report bac over the untruthful report bca . Consequently, the best response dynamic reliably converges to equilibrium assignments but does not reliably converge to truthful preference reports in the strategy proof mechanisms.

4 Experimental Design and Procedures

This study implements a 2x3 experimental design with six experimental treatments illustrated by Table 1. Each column of this table denotes one of three widely employed school choice mechanisms: the deferred acceptance mechanism, the top trading cycles mechanism, and the Boston mechanism. For each of these mechanisms, this study implements two experimental treatment conditions: one with continuous assignment feedback and another with conventional discrete feedback. A total of eighteen experimental sessions were conducted, three for each of the six treatment blocks. Each experimental session was conducted with twenty-four subjects for a total of four hundred and thirty two experimental subjects. Each subject participated in only one experimental session. All sessions were conducted at the Texas A&M Economic Research Laboratory.

	Top Trading Cycles	Deferred Acceptance	Boston
Discrete Feedback	3 Sessions	3 Sessions	3 Sessions
Continuous Feedback	3 Sessions	3 Sessions	3 Sessions

Table 1: 3x2 Experimental Design with Three Sessions Per Block

During each experimental session, subjects were divided into three groups of eight participants. Each group was assigned one of the three student types described in Subsection 3.1. The same priorities and induced values were used across all six experimental treatment conditions. Each experimental session consisted of twelve reporting periods and each reporting period lasted for exactly one minute. At the beginning of each reporting period, subjects were informed about the earnings that they would receive from being assigned each of the three possible options: a , b , or c . This information remained visible to subjects for the duration of the experimental session. To avoid the possibility of introducing any psychological ordering or labeling bias, the labeling for each school and the order in which the options were presented was randomly reassigned at the beginning of each period.

During each period, subjects selected a preference report. Throughout every reporting period in every treatment condition, subjects were free to adjust their preference reports as frequently as desired. At the end of every reporting period, preference reports were finalized and assignments were made based on the finalized preference reports. Figure 3 depicts the experimental interface. Under the discrete feedback treatment, subjects could only observe their assignments at the end of the reporting period, after all preference reports were finalized. Under the continuous feedback treatment, subjects could also observe their assignments under the currently selected preference reports throughout the reporting period. At the end of each session, subjects received their average earnings over all twelve periods plus a five dollar participation bonus.

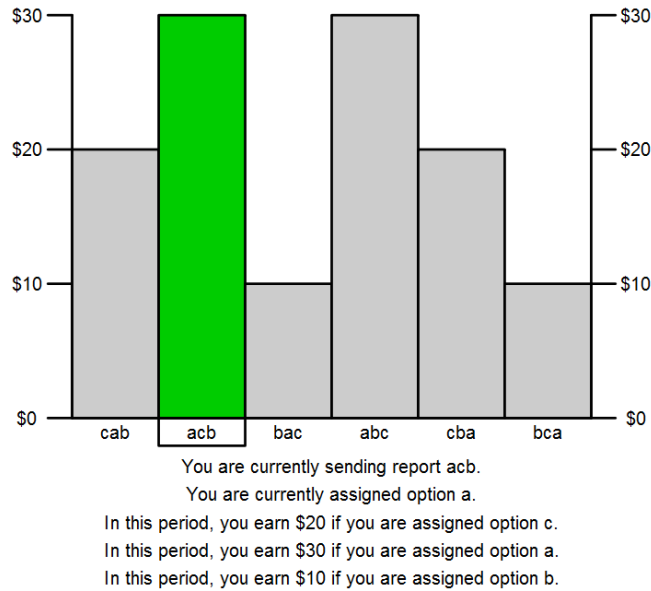


Figure 3: Screenshot of the Experimental Interface

5 Hypotheses

The best response dynamic predicts that continuous assignment feedback will help school choice mechanisms achieve equilibrium assignments by providing more opportunity for learning and adjustment. The iterative adjustment process described by the best response dynamic can only occur when participants receive feedback. Accordingly, the provision of continuous feedback throughout the preference reporting period is hypothesized to significantly increase the proportion of equilibrium assignments under all three mechanisms.

Hypothesis 1. *School choice mechanisms will achieve equilibrium assignments more often when they provide assignment feedback throughout the preference reporting period than when they only provide assignment feedback after all preference reports have been finalized.*

The top trading cycles mechanism is strategy proof and its dominant strategy Nash equilibrium always achieves Pareto efficiency. In the experimental environment, the dominant strategy Nash equilibrium of the top trading cycles mechanism yields an assignment under which all students of types 1 and 2 are

assigned to their most preferred school. School a is the favorite of both type 2 and type 3 students, so it is not possible to assign more than two thirds of the student population to their most preferred school. This theoretical prediction motivates the second hypothesis.

Hypothesis 2. *The top trading cycles mechanism will assign more students their most preferred school when it provides continuous assignment feedback.*

The student optimal deferred acceptance mechanism is strategy proof and its dominant strategy Nash equilibrium always eliminates justified envy. In the experimental environment, its dominant strategy Nash equilibrium yields the unique assignment which completely eliminates justified envy under the true preferences. This theoretical prediction motivates the third hypothesis.

Hypothesis 3. *The deferred acceptance mechanism will eliminate more justified envy when it provides continuous assignment feedback.*

The Boston mechanism's set of Nash equilibrium assignments is equal to the set of assignments that eliminate justified envy. In the experimentally implemented school choice environment, its Nash equilibrium assignment uniquely eliminates justified envy under the true preferences. This theoretical prediction motivates the fourth hypothesis.

Hypothesis 4. *The Boston mechanism will eliminate more justified envy when it provides continuous assignment feedback.*

No school choice mechanism can guarantee both Pareto efficiency and the elimination of justified envy. In the experimental environment, the unique assignment that eliminates justified envy is Pareto dominated and does not give any of the students their most preferred school. Accordingly, we do not hypothesize that continuous assignment feedback will eliminate more justified envy in the top trading cycles mechanism or achieve greater efficiency in the deferred acceptance mechanism.

The best response dynamic predicts that continuous assignment feedback will help school choice mechanisms achieve equilibrium assignments. However, it does not predict that it will help school choice mechanisms induce truthful preference reports. Accordingly, we do not hypothesize that that continuous feedback will increase the proportion of truthful preference reports.

	Feedback		t-test	Rank-Sum Test
	Discrete	Continuous	p-value	p-value
Top Trading Cycles	0.762	0.921	<0.001	
Deferred Acceptance	0.690	0.981	<0.001	<0.001
Boston	0.202	0.892	<0.001	

Table 2: Means and hypothesis tests regarding the proportion of equilibrium assignments. The unit of observation for the t-tests is one period. The unit of observation for the Mann-Whitney-Wilcoxon rank-sum test is one session.

	Feedback		t-test	Rank-Sum Test
	Discrete	Continuous	p-value	p-value
Top Trading Cycles	0.53241	0.62963	<0.001	0.200
Deferred Acceptance	0.82755	0.98843	<0.001	0.004
Boston	0.59259	0.94097	<0.001	

Table 3: Hypothesis tests regarding the elimination of justified envy. The unit of observation for the t-tests is one period. The unit of observation for the Mann-Whitney-Wilcoxon rank-sum test is one session.

	Feedback		t-test	Rank-Sum Test
	Discrete	Continuous	p-value	p-value
Top Trading Cycles	0.48727	0.59722	<0.001	0.100
Deferred Acceptance	0.18750	0.01042	<0.001	0.002
Boston	0.52431	0.06134	<0.001	

Table 4: Hypothesis tests regarding the proportion most preferred assignments. The unit of observation for the t-tests is one period. The unit of observation for the Mann-Whitney-Wilcoxon rank-sum test is one session.

6 Results

Table 2 presents hypothesis tests for the effect of continuous feedback on equilibrium assignments. A non-parametric Mann-Whitney-Wilcoxon rank-sum test finds that equilibrium assignment percentages are significantly higher under continuous feedback at the one percent level. A t-test finds that all three mechanisms achieve equilibrium assignments significantly more often under continuous feedback than under discrete feedback at the one percent level. The adaptive best response dynamic anticipates this result, suggesting that the provision of continuous feedback can help school choice mechanisms more reliably achieve equilibrium assignments by providing more opportunity for learning and adjustment during the preference reporting period.

Result 1. *School choice mechanisms achieved equilibrium assignments significantly more often when participants were provided with assignment feedback throughout the preference reporting period.*

Figure 4 illustrates the empirical assignment dynamics. The horizontal axis of each graph indicates seconds within a preference reporting period. The vertical axis of each graph indicates the average percentage of participants receiving their equilibrium assignment. Under every treatment condition, the final assignments were exclusively based on the preference reports submitted at the end of the period. The first graph illustrates the empirical results under the student optimal deferred acceptance mechanisms. The second graph illustrates the empirical results under the Boston mechanism. The third graph illustrates the empirical results under the top trading cycles mechanism.

In all three graphs, the blue line illustrates the mean path under continuous feedback and the green line illustrates the mean path under conventional discrete feedback. The percentage of equilibrium assignments at the end of the reporting period is consistently higher under continuous feedback. Under discrete feedback implementations of the manipulable Boston mechanism, the percentage of equilibrium assignments decreased over the preference reporting period, suggesting that discrete-feedback was insufficient for many subjects to develop strategic preference reports in the Boston mechanism.

Table 3 presents hypothesis tests for the effect of continuous feedback on the elimination of justified envy. A non-parametric Mann-Whitney-Wilcoxon

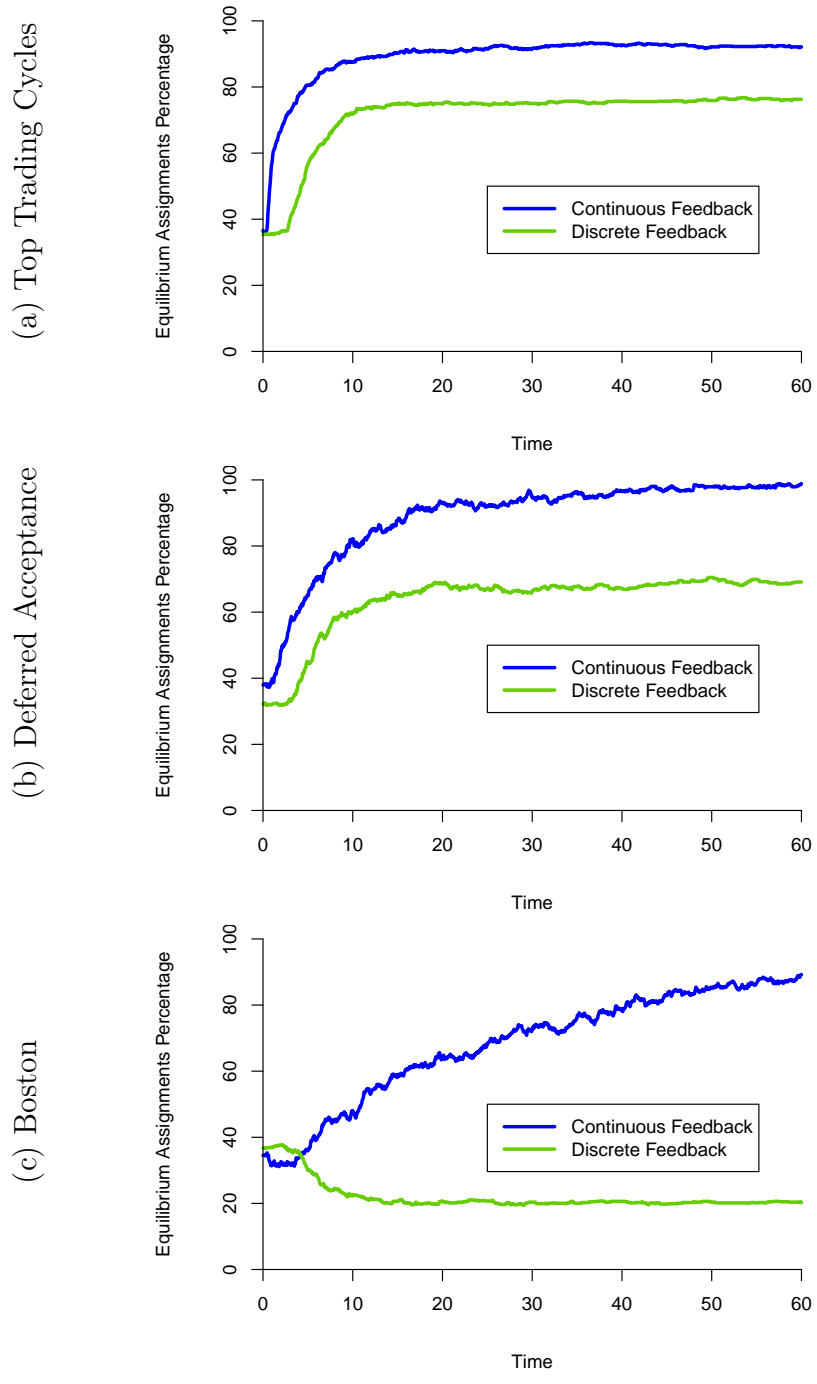


Figure 4: Percentage of equilibrium assignments over the preference reporting period. Sub-figures (a), (b), and (c) illustrate the empirical results under the the top trading cycles, deferred acceptance, and Boston mechanisms respectively.

rank-sum test finds that the deferred acceptance mechanism and the Boston mechanism eliminate significantly more justified envy under continuous assignment feedback at the one percent level. The assignment that uniquely eliminates justified envy in this environment is the equilibrium assignment for both the deferred acceptance mechanism and the Boston mechanism. Hence this result is consistent with the presence of more equilibrium assignments under continuous assignment feedback.

Result 2. *All three school choice mechanisms eliminated significantly more justified envy when subjects received continuous assignment feedback.*

The dominant strategy Nash equilibrium outcome of the top trading cycles mechanism does not fully eliminate justified envy because it gives type 3 students justified envy towards type 2 students. However, it does eliminate justified envy among type 1 and type 2 students. Consistent with this prediction, about two-thirds of subjects of justified envy was eliminated under the top trading cycles mechanism with continuous feedback. In contrast, only 53% of subjects were free from justified envy under the top trading cycles mechanism with conventional discrete feedback.

Since school a is the favorite school for both type 2 and type 3 students, it is not possible to assign more than two thirds of the student population to their most preferred school. Table 4 presents hypothesis tests for the effect of continuous feedback on the percentage of students receiving their most preferred assignment. A non-parametric Mann-Whitney-Wilcoxon rank-sum test finds that the deferred acceptance mechanism and the Boston mechanism were significantly less likely to assign subjects their most preferred option under continuous assignment feedback at the one percent level. A t-test finds that the top trading cycles mechanism assigned subjects their most preferred option significantly more often under continuous feedback than under discrete feedback at the one percent level.

Result 3. *The top trading cycles mechanism gave subjects their most preferred option significantly more often when it provided subjects with continuous assignment feedback.*

School choice mechanisms face a fundamental tradeoff between efficiency and the elimination of justified envy. Eliminating justified envy can sometimes ne-

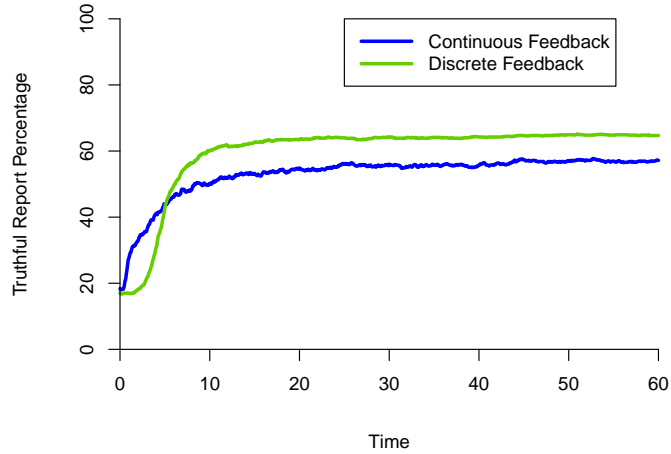
cessitate inefficient assignments. In the experimental environment, the dominant strategy Nash equilibrium assignment of the deferred acceptance mechanism uniquely eliminates justified envy but does not assign any of the participants to their most preferred school. In contrast, the dominant strategy Nash equilibrium assignment of the top trading cycles mechanism is Pareto efficient and assigns both type 2 and type 3 students to their most preferred school. Accordingly, an increase in equilibrium assignments has a different effect on efficiency in different mechanisms.

Figure 5 illustrates the percentage of subjects who accurately reported their preferences over time during a preference reporting period. The first graph illustrates the empirical results under the student optimal deferred acceptance mechanisms. The second graph illustrates the empirical results under the Boston mechanism. The third graph illustrates the empirical results under the top trading cycles mechanism. The vertical axis of each graph indicates the percentage of participants who accurately reported their preferences. The horizontal axis of each graph indicates seconds within the preference reporting period. The blue line illustrates the mean path under continuous feedback and the green line illustrates the mean path under conventional discrete feedback.

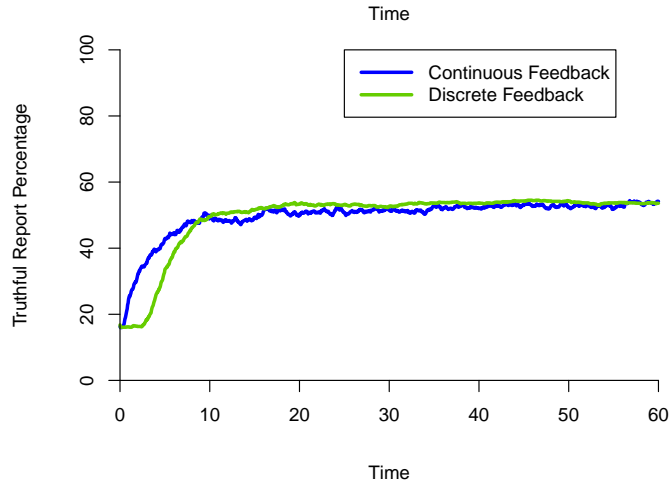
Under the deferred acceptance mechanism, subjects selected truthful preference reports about equally often under continuous and discrete feedback. Under the top trading cycles mechanism subjects were slightly less likely to truthfully report their preferences under continuous assignment feedback. These results are consistent with theoretical predictions of the adaptive best response dynamic. Under both mechanisms, the best response dynamic converges to equilibrium assignments, but does not converge to truthful preference reports. In line with these predictions, subjects were significantly more likely to achieve equilibrium assignments when they received assignment feedback during the preference reporting period, but they were not significantly more likely to report their preferences accurately.

Less than one-fourth of the subjects in the manipulable Boston mechanism selected truthful preference reports under continuous assignment feedback. Yet over 80% of these subjects achieved equilibrium assignments. The decrease in truthful reporting is a direct result of the increase in optimal preference reports as participants generally select untruthful preference reports in Nash equilibria

(c) Top Trading Cycles



(a) Deferred Acceptance



(b) Boston

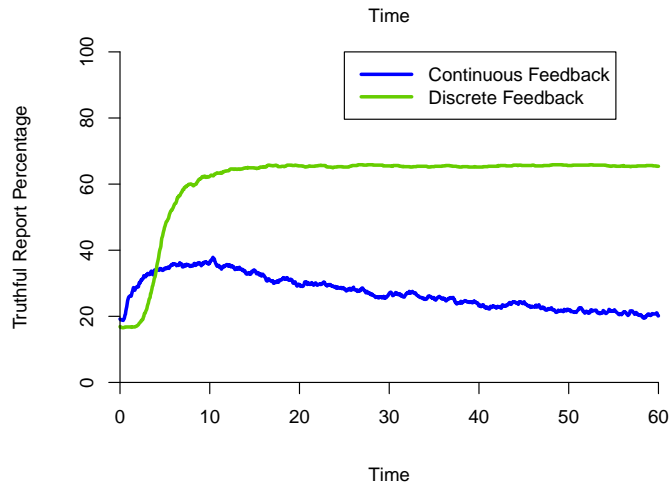


Figure 5: Percentage of truthful preference reports over time within a period. The first graph illustrated the empirical results under the student optimal deferred acceptance mechanisms. The second graph illustrates the empirical results under the Boston mechanism. The third graph illustrates the empirical results under the top trading cycles mechanism.

of the Boston mechanism. In contrast about two-thirds of subjects in the Boston mechanism naively selected truthful preference reports under discrete feedback, suggesting that many subjects selected their preference reports non-strategically under discrete feedback.

As predicted by the best response dynamic, subjects were significantly more likely to achieve equilibrium assignments when they received assignment feedback during the preference reporting period, but they were not significantly more likely to report their preferences accurately. While truthful reports are always optimal in strategy proof mechanisms, optimal preference reports are not always truthful. In equilibrium, the top trading cycles mechanism achieves Pareto efficiency. Accordingly, when subjects received continuous feedback, the top trading cycles mechanism achieved greater efficiency. In equilibrium, the deferred acceptance mechanism and the Boston mechanism eliminate justified envy. Accordingly, when subjects received continuous assignment feedback, both of these mechanisms eliminated more justified envy.

7 Conclusion

Classical mechanism design theory often operates under the assumption that strategy proof mechanisms will induce truthful preference reports. This assumption is often difficult to verify in the field where preferences are rarely observable and mechanisms rarely satisfy the exact assumptions of theory. Experimental investigations can test mechanism design theory in a controlled environment. Previous studies find that even strategy proof mechanisms fail to reliably achieve equilibrium outcomes. The best response dynamic predicts that the provision of assignment feedback throughout the preference reporting period can help school choice mechanisms more reliably achieve equilibrium outcomes by providing more opportunities for learning and adjustment. To test this hypothesis, the experiment investigates novel continuous feedback implementations of school choice mechanisms.

Consistent with theoretical predictions of the best response dynamic, experimental subjects achieved equilibrium assignments significantly more often under continuous feedback than under conventional discrete feedback. In the

top trading cycles mechanism, subjects were significantly more likely to receive their most preferred school under continuous assignment feedback. In the deferred acceptance mechanism and the Boston mechanism, subjects were significantly less likely to exhibit justified envy under continuous assignment feedback. While truthful reports are always optimal in strategy proof mechanisms, optimal preference reports are not always truthful. Accordingly, continuous feedback implementations of strategy proof mechanisms achieved significantly more equilibrium assignments, but they did not exhibit significantly more truthful preference reports.

Student assignment mechanisms impact the well being of children in many school districts throughout the world. The Boston mechanism was originally used in Boston's school choice system. The New Orleans recovery school district used an algorithm based on the top trading cycles assignment mechanism (Vanacore, 2012). A variation of the student optimal deferred acceptance mechanism was employed in New York City (Roth, 2008). The experimental results of this study suggest that the provision of assignment feedback throughout the preference reporting period can help school choice mechanisms more reliably achieve policy goals. Further research is necessary to determine the extent to which these findings generalize to field settings.

References

- Atila Abdulkadiroglu and Tayfun Sönmez. School choice: A mechanism design approach. *The American Economic Review*, 93(3):729–747, 2003.
- Atila Abdulkadiroglu, Parag Pathak, Alvin E Roth, and Tayfun Sonmez. Changing the boston school choice mechanism. Technical report, National Bureau of Economic Research, 2006.
- George W Brown. Iterative solution of games by fictitious play. *Activity analysis of production and allocation*, 13(1):374–376, 1951.
- Timothy Cason, Daniel Friedman, and Ed Hopkins. Cycles and instability in a rock-paper-scissors population game: A continuous time experiment. *Review of Economic Studies*, 2013.

- Yan Chen and Tayfun Sönmez. School choice: an experimental study. *Journal of Economic theory*, 127(1):202–231, 2006.
- Umut Dur, Robert G Hammond, and Thayer Morrill. Identifying the harm of manipulable school-choice mechanisms. Technical report, mimeo, 2015.
- Haluk Ergin and Tayfun Sönmez. Games of school choice under the boston mechanism. *Journal of public Economics*, 90(1):215–237, 2006.
- David Gale and Lloyd S Shapley. College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15, 1962.
- Itzhak Gilboa and Akihiko Matsui. Social stability and equilibrium. *Econometrica: Journal of the Econometric Society*, pages 859–867, 1991.
- Binglin Gong and Yingzhi Liang. A dynamic college admission mechanism in inner mongolia: Theory and experiment. *Working Paper*, 2016.
- Ryan Oprea, Keith Henwood, and Daniel Friedman. Separating the hawks from the doves: Evidence from continuous time laboratory games. *Journal of Economic Theory*, 146(6):2206–2225, 2011.
- Joana Pais and Ágnes Pintér. School choice and information: An experimental study on matching mechanisms. *Games and Economic Behavior*, 64(1):303–328, 2008.
- Alvin E Roth. The economics of matching: Stability and incentives. *Mathematics of operations research*, 7(4):617–628, 1982.
- Alvin E Roth. Deferred acceptance algorithms: History, theory, practice, and open questions. *international Journal of game Theory*, 36(3-4):537–569, 2008.
- Lloyd Shapley and Herbert Scarf. On cores and indivisibility. *Journal of mathematical economics*, 1(1):23–37, 1974.
- Daniel Stephenson. Coordination and evolutionary dynamics: When are evolutionary models reliable? *Games and Economic Behavior*, 113:381–395, 2019.

A Vanacore. Centralized enrollment in recovery school district gets first tryout.
Times-Picayune, April, 16, 2012.